10\19\09

Duality in Linear Programming

Primal

$$\min \quad \underset{\sim}{c}_1 \underset{\sim}{x}_1 + c_2 \underset{\sim}{x}_2 + \underset{\sim}{c}_3 \cdot \underset{\sim}{x}_3$$

with $\underset{\sim}{x}_1 \geq 0$ and $\underset{\sim}{x}_2 \leq 0$

$$A_{1,1} \underset{\sim}{x}_1 + A_{1,2} \underset{\sim}{x}_2 + A_{1,3} \underset{\sim}{x}_3 \geq \underset{\sim}{b}_1$$

$$A_{2,1} \underset{\sim}{x} + A_{2,2} \underset{\sim}{x}_2 + A_{2,3} \underset{\sim}{x}_3 \leq \underset{\sim}{b}_2$$

$$A_{3,1} \underset{\sim}{x}_1 + A_{3,2} \underset{\sim}{x}_2 + A_{3,3} \underset{\sim}{x}_3 = \underset{\sim}{b}_3$$

Primal

$$\min \; c_1 \overset{\ge 0}{x_1} + c_2 \overset{\le 0}{x_2} + c_3 \cdot x_3$$

$$A_{1,1} x_1 + A_{1,2} x_2 + A_{1,3} x_3 \ge b_1$$

$$A_{2,1} x + A_{2,2} x_2 + A_{2,3} x_3 \le b_2$$

$$A_{3,1} x_1 + A_{3,2} x_2 + A_{3,3} x_3 = b_3$$

$$\begin{array}{c|ccc|c}
 & c_1 & c_2 & c_3 & \\
\hline
y_1 & A_{1,1} & A_{1,2} & A_{1,3} & b_1 \\
y_2 & A_{2,1} & A_{2,2} & A_{3,2} & b_2 \\
y_3 & A_{3,1} & A_{3,2} & A_{3,3} & b_3 \\
\hline
 & x_1 & x_2 & x_3 &
\end{array}$$

Dual

$$\max \; b_1 \cdot \overset{\ge 0}{y_1} + b_2 \cdot \overset{\le 0}{y_2} + b_3 y_3$$

$$A^T_{1,1} y_1 + A^T_{2,1} y_2 + A^T_{3,1} y_3 \le c_1$$

$$A^T_{1,2} y_1 + A^T_{2,2} y_2 + A^T_{3,2} y_3 \ge c_2$$

$$A^T_{1,3} y_1 + A^T_{2,3} y_2 + A^T_{3,3} y_3 = c_3$$

$$P_A = \max \xi$$

$$\xi - \sum_{i=1}^{m} a_{ij} p_i \leq 0 \qquad j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m} p_i = 1$$

$$p_i \geq 0 \qquad i = 1, \ldots, m$$

$$P_B = \min \eta$$

$$\eta - \sum_{j=1}^{n} a_{ij} q_j \geq 0 \qquad i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{n} q_j = 1$$

$$q_j \geq 0 \qquad j = 1, 2, \ldots, n$$

$$\rho_A = \max \xi$$

y_1 $$\xi - \sum_{i=1}^{m} a_{ij} p_i \leq 0 \qquad j=1,2,\dots,n$$
$\vdots$
y_n

y_0 $$\sum_{i=1}^{m} p_i = 1$$

$$p_i \geq 0 \qquad i=1,\dots,m$$

Dual

min y_0

$$\begin{array}{c|cc}
 & 0 & \\
p_i & -a_{i1} & y_1 \\
 & -a_{i2} & y_2 \\
 & \vdots & \vdots \\
 & -a_{in} & y_n \\
 & 1 & y_0
\end{array}$$

$$y_1 + \dots + y_n = 1$$

$$y_0 - \sum_j a_{ij} y_j \geq 0$$

$$y_1, \dots, y_n \geq 0$$

Dominance

$$\begin{bmatrix} 3 & 6 & 4 & 7 & 9 \\ 5 & 6 & 5 & 8 & 10 \\ 3 & 2 & 4 & 5 & 12 \\ 6 & 3 & 1 & 4 & 7 \end{bmatrix}$$

A will never play 1

2 dominates 1 for A

B will never play 5

4 dominates 5 for B

Symmetric Game

$$\underbrace{\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}}_{A}$$

$$A^T = -A$$

A is anti-symmetric

Value of game $= 0$

(1) $\mathrm{PAY}(\underline{p}, \underline{p}) = 0, \quad \forall\ \underline{p}$

$$\sum a_{ij}\, p_i p_j = 0$$

$a_{ii} = 0$ & $a_{ij} + a_{ji} = 0$

$$\mathrm{PAY} \begin{bmatrix} 0 & & & & & & & \\ & 0 & & & & & & \\ & & 0 & & & & & \\ & & & 0 & & & & \\ & & & & 0 & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & 0 \end{bmatrix}$$

$P_A \leq 0$

$P_B \geq 0$

$$0 \geq \underbrace{P_A = P_B}_{\geq 0} \geq 0$$