10\12\09

2 person, zero sum games

$$\begin{array}{cc} & j \\ i & \begin{bmatrix} & & \\ & a_{ij} & \\ & & \end{bmatrix} \end{array}$$

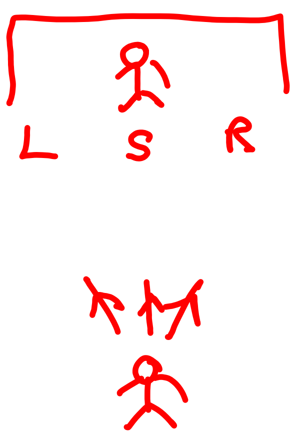
2 players A, B

A chooses i, B chooses j (without each other's knowledge)
B pays A, a_{ij}

Rock, Paper, Scissors

$$\begin{array}{c c} & \begin{array}{ccc} R & P & S \end{array} \\ \begin{array}{c} R \\ P \\ S \end{array} & \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \end{array}$$

Penalty Kicks in Soccer



← keeper

$$\begin{array}{c c} & \begin{array}{ccc} L & C & R \end{array} \\ \begin{array}{c} L \\ C \\ R \end{array} & \begin{bmatrix} -2 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & -2 \end{bmatrix} \end{array}$$

shooter

Games arise anywhere there is conflict: economics, war, animal behavior.

Assume an unending sequence of <u>plays</u> of the game

Strategy is some rule for playing the game.

$S_A = \{ A\text{'s strategies} \}$

$S_B = \{ B\text{'s strategies} \}$

Pure strategy for A : (i)

Always play i

We expand set of strategies beyond just pure strategies

$u \in S_A$

$v \in S_B$

$PAY(u, v) =$ average payoff to A

$P((i), (j)) = a_{ij}$

Two ways of viewing:

(1) Stable Solution:

(u_0, v_0) is stable if

$$PAY(u, v_0) \leq PAY(u_0, v_0) \leq PAY(u_0, v)$$

?? Does $\exists$ a stable solution ??

If $S_A = \{(1), (2), \ldots (n)\}$

& $S_B = \{(1), (2), \ldots (n)\}$

then there may not be a stable solution.

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

(ii) Both players are cautious:

If A plays u, can definitely get

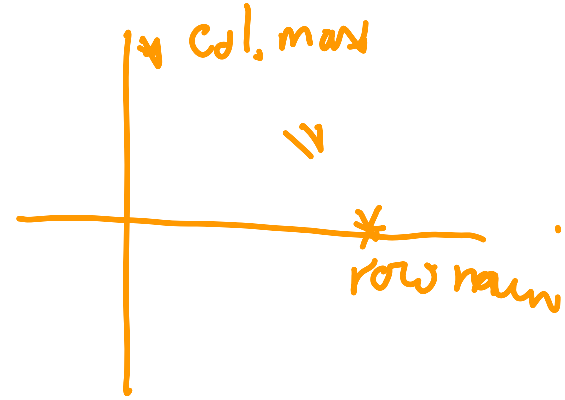
$$\text{ROWMIN}(u) = \min_v \text{PAY}(u,v)$$

A chooses $\max_u \text{ROWMIN}(u)$. $\leftarrow P_A$

B chooses $\min_v \text{COLMAX}(v) \leftarrow P_B$

Both notions lead to same strategy here.

$$P_A \leq P_B$$



We show next

$$P_A = P_B \iff \exists \text{ stable solution.}$$

$$\begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 8 \\ 1 & -7 & -7 \end{bmatrix}$$

<u>Mixed Strategy</u>

A chooses vector $p = p_1, p_2, \dots p_m \geq 0$, $p_1 + \dots + p_m = 1$

B chooses a vector $q = q_1, q_2, \dots, q_n \geq 0$, $q_1 + \dots + q_n = 1$

Next play for A is i with prob. p_i, etc.

$$\mathrm{PAY}(\underline{p}, \underline{q}) = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} p_i q_j$$

$$S_A = \{ \underline{p} \}$$

$$S_B = \{ \underline{q} \}$$