Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 4: Due Wednesday October 29.

1. Find the optimal ordering strategy for the following inventory system. If you order an amount Q, it costs $AQ^{1/2}$ and the inventory cost is I per unit per period. The demand is λ units per period and no stock-outs are allowed.

   Solution: Suppose that the order quantity is Q. Then the ordering cost per period is $\frac{AQ^{1/2}}{T}$ where $T = Q/\lambda$. Thus the average total cost per period is
   $$\frac{A\lambda}{Q^{1/2}} + \frac{IQ}{2}.$$
   This is minimised when $Q = (A\lambda/I)^{2/3}$, giving an optimal cost of $3(A^2\lambda^2 I)^{1/3}/2$.

2. 16.6-5 of book.

3. 19.5-7 of book.

16.6-5

(a) Transition Matrix, $P = \begin{bmatrix} 0 & 0.875 & 0.062 & 0.062 \\ 0 & 0.75 & 0.125 & 0.125 \\ 0 & 0 & 0.5 & 0.5 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Solution:

$\pi_1 = 0.154$
$\pi_2 = 0.538$
$\pi_3 = 0.154$
$\pi_4 = 0.154$

(b) $\pi \cdot C = 1(0.538) + 3(0.154) + 6(0.154) = \$1923.08$

(c)
$$\begin{cases} \mu_{00} = 1 + 0.875\mu_{10} + 0.0625\mu_{20} + 0.0625\mu_{30} \\ \mu_{10} = 1 + 0.75\mu_{10} + 0.125\mu_{20} + 0.125\mu_{30} \\ \mu_{20} = 1 + 0.5\mu_{20} + 0.5\mu_{30} \\ \mu_{30} = 0 + 1 \end{cases}$$

$\Rightarrow \mu_{00} = 6.5$, which is expected recurren time for state 0.

19.5-7 (a) Let state 0 = chemical 0 produced this month
1 = chemical 1 produced this month

decision 1 = use process A next month
2 = use process B next month

The four stationary deterministic policies are:

i	$d_i(R_1)$	$d_i(R_2)$	$d_i(R_3)$	$d_i(R_4)$
0	1	1	2	2
1	1	2	1	2

(b) Markovian Decision Processes Model:

Number of states = 2

Number of decisions = 2

Cost Matrix, C(ik):

$\begin{bmatrix} 28 & 26 \\ 37 & 24 \end{bmatrix}$

Transition Matrix, p(ij)[2]:

$\begin{bmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{bmatrix}$

Transition Matrix, p(ij)[1]:

$\begin{bmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{bmatrix}$

Initial Policy:

d0(R1) = 2
d1(R1) = 2

Discounted Cost Policy Improvement Algorithm:

Discount Factor = 0.5

ITERATION # 1

Value Determination:

g(R1) = 26 + (0.5) [0.2V0(R1) + 0.8V1(R1)]
g(R1) = 24 + (0.5) [0.3V0(R1) + 0.7V1(R1)]

Solution of Value Determination Equations:
V1(R1) = 50.48
V2(R1) = 48.57

19.5-7. (continued)

Policy Improvement:

State 0:
28 + (0.5) [0.2 (50.48) + 0.8 (48.57)] = 52.48
26 + (0.5) [0.2 (50.48) + 0.8 (48.57)] = 50.48

State 1:
37 + (0.5) [0.3 (50.48) + 0.7 (48.57)] = 61.57
24 + (0.5) [0.3 (50.48) + 0.7 (48.57)] = 48.57

Optimal Policy:
d0(R2) = 2
d1(R2) = 2

V0(R1) = 50.48
V1(R1) = 48.57