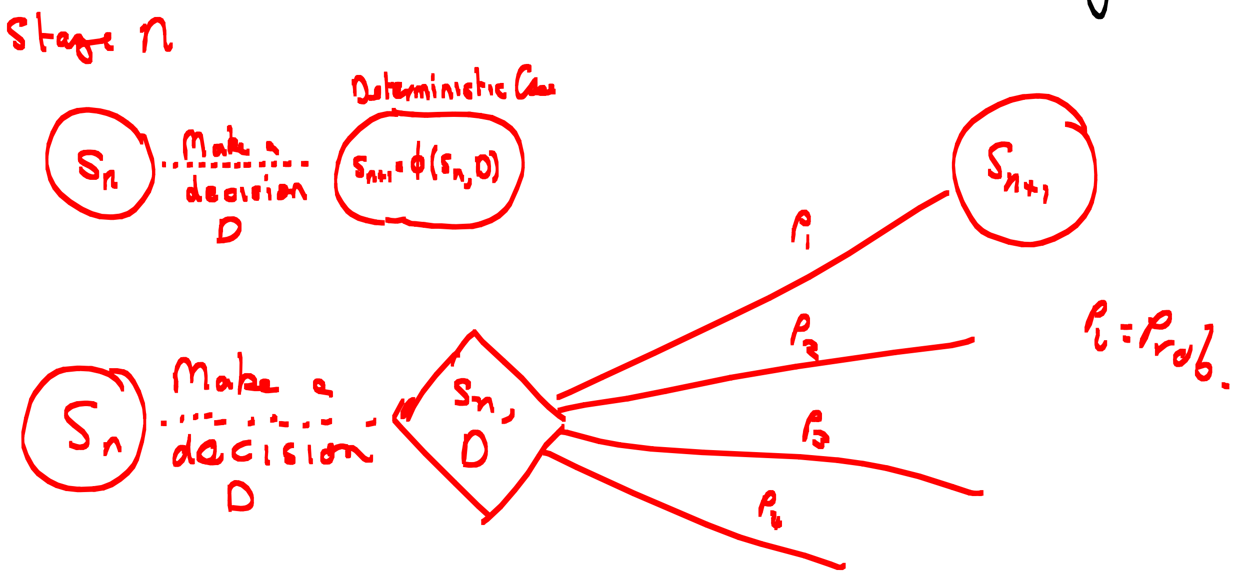
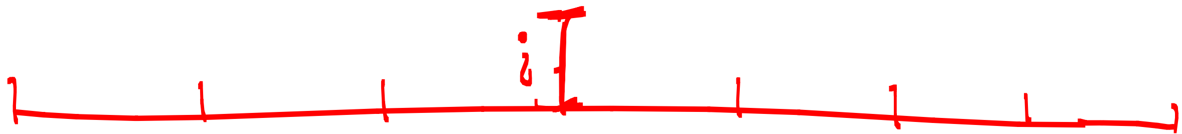
9/3/08

Probabilistic Dynamic Programming



Production Problem



i Decision = x

inventory

$i, x \xrightarrow{\text{Random Deman}}$ new state.

Objective : (a) Max. Prob. of meeting demand or (b) maximise expected profit

Minimise expected cost.

Pay a penalty for lost demand, π per unit.

d_n = demand in period n $\quad P(n,d) = P[d_n = d]$

$$f_n(i) = \min_x \left[c(x) + \sum_d P(n,d) \left[\pi (d-i-x)^+ + f_{n+1}\left((x+i-d)^+\right) \right] \right]$$

minimum expected cost of proceeding for $n, n+1, \ldots$ starting with i in stock

Choose x before we know d.

$$f_n(i) = \sum_d P(n,d) \min_x \left(c(x) + \pi (d-i-x)^+ + f_{n+1}((x+i-d)^+) \right)$$

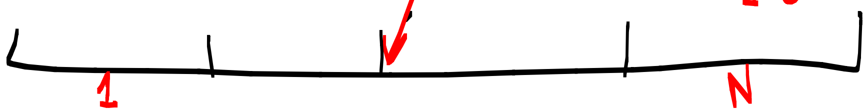
Make decision, after we know the demand.

Example 6

decide to make x

Cost: $= A + Bx \quad x > 0$

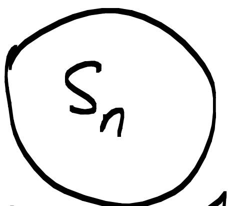
$= 0 \quad x = 0$



$C > 0$ cost of failure

Trying to make one of something difficult.

Prob[an item is defective] $= \frac{1}{2}$



0 Made 1

1 Need to make ≥ 1

$f_{N+1}(1) = C$

$$f_n(0) = 0$$

$$f_n(1) = \min\left\{ f_{n+1}(1),\ \min_{x>0}\left[A + Bx + \left(\tfrac{1}{2}\right)^x f_{n+1}(1)\right]\right\}$$