## 9\22\08

(i) A conservative A tries to maximise his\her minimum winnings.
A conservative B tries to minimise his\her maximum loss

This may not be "stable".

(ii) In a stable solution (Nash Equilibrium) neither player has an incentive to make a unilateral change.

$$P_A = \max_u \min_v PAY(u,v)$$

$$P_B = \min_v \max_u PAY(u,v)$$

<u>Thm</u>

(I) $P_A \leq P_B$

(II) $\exists$ stable solution $\Longleftrightarrow P_A = P_B$

(I) $\mathrm{ROWMIN}(u) = \min_{v} \mathrm{PAY}(u,v)$

$\leq \mathrm{COLMAX}(v)$

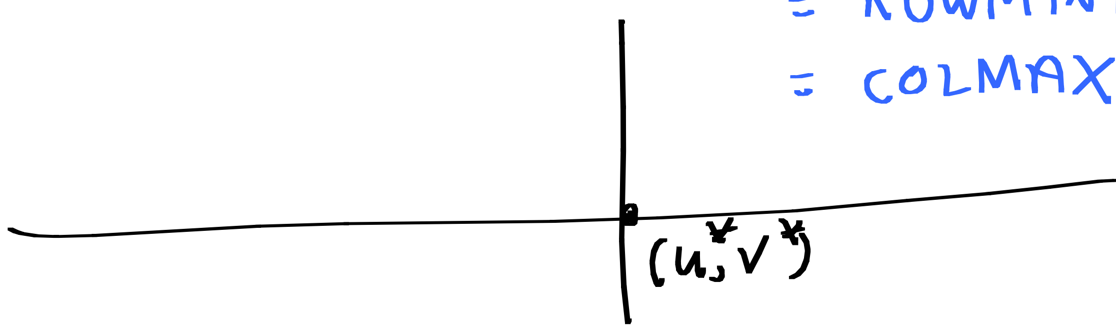
$\forall u, v.$

$\Rightarrow P_A \leq P_B$

$\max_u \mathrm{ROWMIN}(u)$ $\quad$ $\min_v \mathrm{COLMAX}(v)$

(II) $(u^*, v^*)$ is stable

$\mathrm{PAY}(u^*, v^*)$
$= \mathrm{ROWMIN}(u^*)$
$= \mathrm{COLMAX}(v^*)$

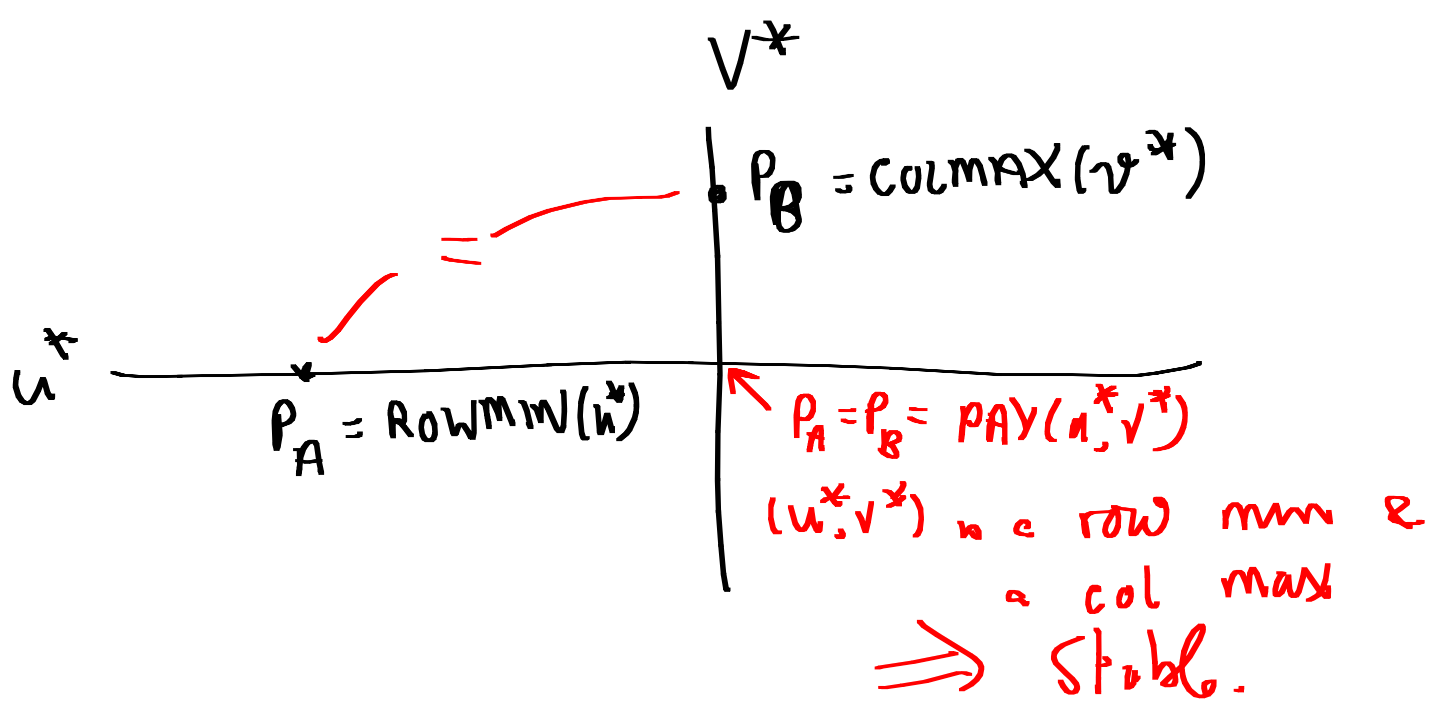


$$
\begin{aligned}
P_A &= \max_{u} \mathrm{ROWMIN}(u) \\
&\geq \mathrm{ROWMIN}(u^*) \\
&= \mathrm{COLMAX}(v^*) \\
&\geq \min_{v} \mathrm{COLMAX}(v) \\
&= P_B
\end{aligned}
$$

(II) Suppose $P_A = \text{ROWMIN}(u^*)$

$= \text{COLMAX}(v^*) = P_B$

$\Rightarrow (u^*, v^*)$ is stable.



$A \searrow \begin{bmatrix} \quad & \quad \end{bmatrix}$

Suppose when restricted to "pure" strategies

$u = (i, i, i, i, i \ldots)$ etc

$v = (i, i, \ldots)$

Suppose $P_A < P_B$

No stable solution with pure strategies

# Mixed Strategy

$A$ is $m \times n$

$p = (p_1, p_2, \ldots, p_m) \geq 0$, $p_1 + \cdots + p_m = 1$

is "mixed strategy" for $A$.

$q = (q_1, q_2, \ldots, q_n) \geq 0$ & $q_1 + q_2 + \cdots + q_n = 1$

is a "mixed strategy" for $B$.

$\mathrm{PAY}(p, q)$ = expected winnings by $A$ each round

$= \sum_{i=1}^{m} \sum_{j=1}^{n} A_{ij} \, p_i \, q_j$

$\exists$ stable solution

i.e. $P_A = P_B$ in this set-up.