9/08/08

Integer Programming

$$\min \quad \underline{c}^T \underline{x}$$

$$A\underline{x} = b$$

$$\underline{x} \geq 0$$

$$x_j \text{ integer for } j \in I \subseteq [n]$$

i.e. LP where some variables have to be integral.

Maximize $3x_1 + 2x_2$

$x_1 + x_2 \leq 10$

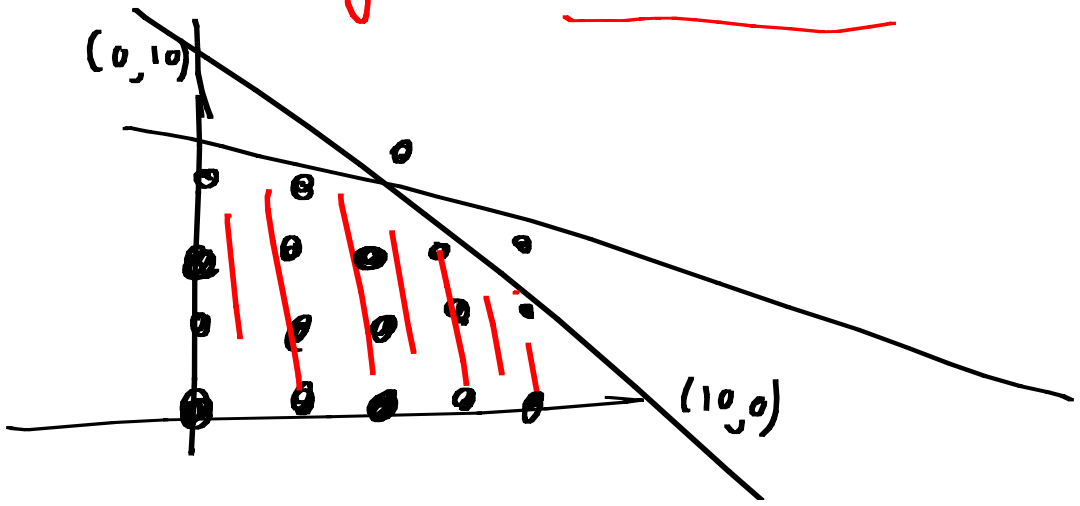
$2x_1 + 3x_2 \leq 7$

$x_1, x_2 \geq 0$ & integer.

No fast, general method.

No algorithm with a good worst-case guarantee.

Feasible Region



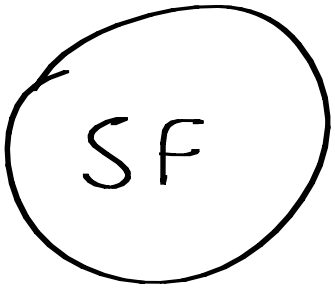
Typical integer variable x_j

takes only 2 values

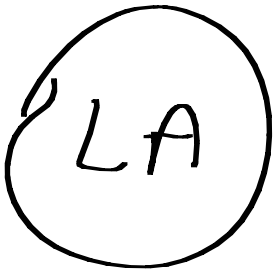
$$x_j = \begin{cases} 0 & \neg \text{ do something} \\ 1 & \text{do something} \end{cases}$$

Model a binary choice as a 0/1 variable.

Company



Decide where / whether to build factory or warehouse.



Capital Available
10

	Profit	Capital Needed	
Build F. in LA	9	6	x_1
" " " SF	5	3	x_2
" W " LA	6	5	x_3
" " " SF	4	2	x_4

0/1

Maximise $Z = 9x_1 + 5x_2 + 6x_3 + 4x_4$

profit from factory in LA

$6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 10$

capital expenditure

Only build one warehouse $x_3 + x_4 \leq 1$

$-x_1 + x_3 \leq 0$

If you build a warehouse LA, must build F.

$-x_2 + x_4 \leq 0$

$x_j = 0$ or 1 $\quad j = 1, 2, 3, 4$ OR $0 \leq x_j \leq 1$ & integer $j = 1, 2, 3, 4$

Set Covering Problem

Flights to be covered	Crew Assignment 1											12
SF → LA	1											
SF → Denver												1
SF → Seattle												3
LA → Chicago		2										
LA → SF												4
Chicago → Seattle												
Seattle → SF												5
Seattle → LA												2
Cost		2										9

S = set of things which must be done.

$|S| = n$

= { flights }

$S_i \subseteq S$ is a possible assignment of tasks

= a crew assignment

$S_1, S_2, \ldots, S_m$

Cost c_j assoc. with "using" S_j

Problem: Choose a collection $S_i, i \in I \subseteq [n]$ which cover S, which minimises total cost.