9\05\08.

10.4.1

Backgammon player.

3 consecutive matches

Begins with \$75

50 : 50 chance of winning a game.

If his money at start of game is M

he can bet $0 \leq x \leq M$

$$\text{Bet } x : M \longrightarrow \begin{cases} M + x & \text{win} \\ M - x & \text{loss} \end{cases}$$

Find strategy to maximise probability of
= \$100 at end

3 stages.

Stage n: $f_n(\overset{\text{state}}{\downarrow}{M}) =$ max. prob. of achieving goal

$$f_4(M) = \begin{cases} 1 & M = 100 \\ 0 & M \neq 100 \end{cases}$$

$$f_n(M) = \max_{0 \leq x \leq M} \left[\frac{1}{2} f_{n+1}(M+x) + \frac{1}{2} f_{n+1}(M-x) \right]$$

$$f_4(M) = \begin{cases} 1 & M = 100 \\ 0 & M \neq 100 \end{cases}$$

$$f_n(M) = \max_{0 \le x \le M} \left[\frac{1}{2} f_{n+1}(M+x) + \frac{1}{2} f_{n+1}(M-x) \right]$$

$$f_3(M) = \max_{x} \left[\frac{f_4(M+x) + f_4(M-x)}{2} \right]$$

$$\therefore \; = \begin{array}{lll} \frac{1}{2} & M > 100 & x = M - 100 \\ 1 & M = 100 & x = 0 \\ \frac{1}{2} & 50 \le M < 100 & x = 100 - M \\ 0 & M < 50 & \end{array}$$

$$f_3(M) = \begin{cases} \frac{1}{2} & M > 100 & x = M-100 \\ 1 & M = 100 & x = 0 \\ \frac{1}{2} & 50 \le M < 100 & x = 100-M \\ 0 & M < 50 & \end{cases}$$

$f_2(M)$

(i) $M > 100$: $f_2(M) = \max_x \dfrac{\overset{1/2}{\cancel{f(M+x)}} + f(M-x)}{2}$

$= \dfrac{\frac{1}{2} + 1}{2} = \frac{3}{4}$

$x = M-100$

(ii) $M = 100$ $\quad f_2(M) = 1$ $\quad x = 0$

(iii) $M < 25$ $\quad f_2(M) = \quad 0$

(iv) $25 \le M < 50$ $\quad f_2(M) = \max_x \dfrac{f_3(M+x) + \overset{0}{\cancel{f_3(M-x)}}}{2}$

$= \quad \frac{1}{4} \quad x = 50-M$

(v) $50 \le M < 75$ $\quad f_2(M) = \max_x \frac{1}{2}\left(f_3(M+x) + f_3(M-x)\right) \quad \frac{1}{2}$

$$f_2(M) = \begin{cases} \frac{3}{4} & M > 100 \\ 1 & M = 100 \\ 3/4 & 75 \le M < 100 \\ 1/2 & 50 \le M < 75 \\ 1/4 & 25 \le M < 50 \\ 0 & M < 25 \end{cases}$$

$$f_1(75) = \max_{x} \frac{f_2(75+x) + f_2(75-x)}{2}$$

$x = 25 \quad 3/4$