8/29/08

Maximise $\quad p_1(x_1) + p_2(x_2) + \cdots + p(x_n)$

$$x_1 + x_2 + \cdots + x_n = M$$

$$x_1, x_2, \ldots, x_n \geq 0 \text{ \& integer}$$

x_i = # medical teams allocated to county i

$p_i(x_i)$ = additional person-years of life, if we put x_i into county i

DP paradigm: I make a sequence of decisions.

$D_1, D_2, \ldots$

Once D_1, I get a smaller problem.

$D_1 \equiv$ what is α_1

$D_2 \equiv$ what is α_2

⋮

Find best x_1.

$$\text{Maximize}_{0 \le x_1 \le M} \quad p_1(x_1) + \left\{ \begin{array}{l} \text{Maximum} \\ p_2(x_2) + \cdots + p_n(x_n) \\ x_2 + \cdots + x_n = L - x_1 \\ x_2, \ldots, x_n \ge 0 \text{ \& integer} \end{array} \right.$$

$$= f_2(M - x_1)$$

$$\max_{0 \le x_1 \le L} \quad p_1(x_1) + f_2(M - x_1)$$

...

How to compute f_2 ?

$L - x_1$

$$f_2(R) = \max_{0 \le x_2 \le R} \; p_2(x_2) + \underbrace{\{\quad\}}_{f_3(R - x_2)}$$

General Recurrence:

$$f_k(R) = \max_{0 \le x_k \le R} \left[p_k(x_k) + f_{k+1}(R - x_k) \right]$$

$k = 1, 2, \ldots, n$

General Recurrence:

$$f_k(R) = \max_{0 \le x_k \le R} \left[p_k(x_k) + f_{k+1}(R - x_k) \right]$$

$k = 1, 2, \ldots, n$

(i) $R = 0, 1, 2, \ldots M$

(ii) $f_n(x_n) = p_n(x_n)$ or $f_{n+1}(x_{n+1}) = 0$

Compute $f_k(R)$ in order $k = n, n-1, n-2, \ldots, 1$

$n = 4$
$R = 6$

	1	2	3	4
0	0	0	0	0
1	4	4	5	2
2	5	5	12	4
3	8	(6)	13	6
4	12	8	13	8
5	13	10	13	10
6	20	11	13	14

$(6) = P_2(3)$

n	x_1 (1)	f_1	x_2 (2)	f_2	x_3 (3)	f_3	x_4 (4)	f_4
0	0	0	0	0	0	0	0	0
1			0	5	1	5	1	2
2			0	12	2	12	(2)	4
R = 3			1	16	2	14	3	6
4			1	18	(2)	16	4	8
5			(1)	20	2	18	5	10
6	(1)	24	1	22	2	20	6	14

$x_2 =$	0	1	2	3	4	5	6
	20	4+18	5+16	6+14	8+12	10+5	11