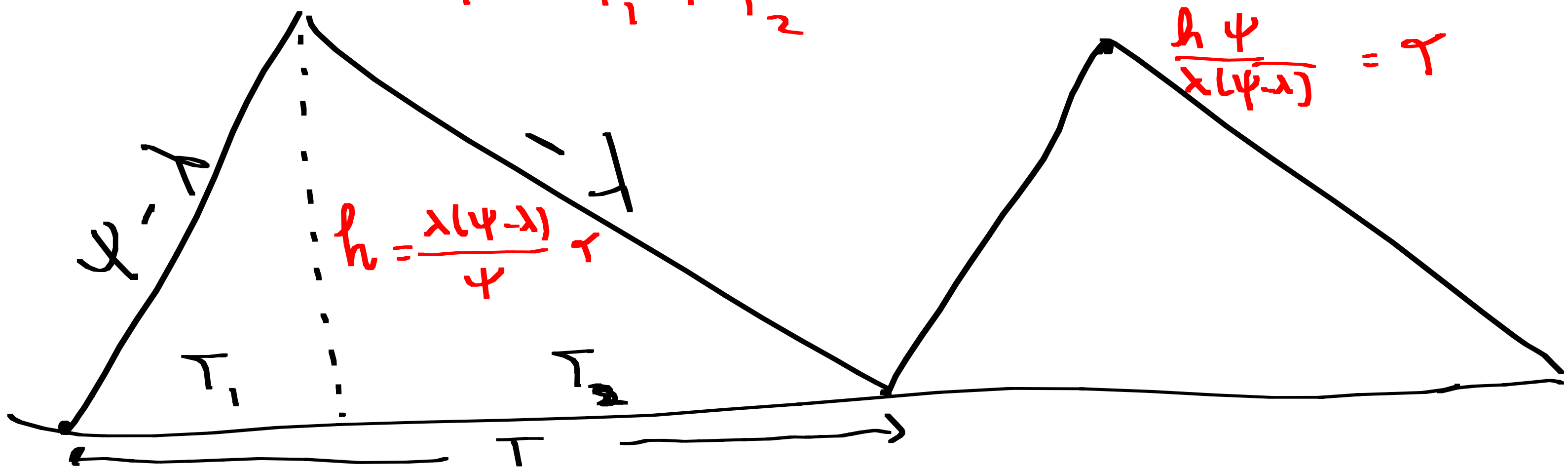
$T_1 = \frac{h}{\psi - \lambda}$ $T_2 = \frac{h}{\lambda}$

$h = (\psi - \lambda) T_1 = \lambda T_2$

$\frac{h}{\psi - \lambda} + \frac{h}{\lambda} = T$

$T = T_1 + T_2$

$\frac{h\psi}{\lambda(\psi - \lambda)} = T$



Produce at rate ψ

I = inventory cost

A = order cost

Cost = Ordering Cost + Inventory Cost

$\frac{A}{T} + \frac{1}{2} I h$

$$\text{Cost} = \frac{A}{T} + \frac{1}{2} I \frac{\lambda(\psi-\lambda)}{\psi} T$$

$$T^* = \left(\frac{2A\psi}{I\lambda(\psi-\lambda)} \right)^{1/2}$$

$$Q^* = \lambda T^* = \left(\frac{2A\psi\lambda}{I(\psi-\lambda)} \right)^{1/2}$$

$$K^* = \left(\frac{2AI\lambda(\psi-\lambda)}{\psi} \right)^{1/2}$$

NewsVendor Problem

(Newsboy problem)

D = demand

S = # ordered

I = initial inventory

K = cost of doing something

c = purchase cost

p = penalty for missed sales

h = over-stocking fee.

Assumption 1

$K = 0$

$I = 0$

$$C(D,S) = cS + p(D-S)^+ + h(S-D)^+$$

cost if demand is D and S was ordered

$x^+ = \max\{0, x\}$

Choose S to minimize $E(C(D,S))$

$f(x) = P_r(D = x)$ (density of demand)

$F(x) = P_r(D \leq x)$

$$\underbrace{E(C(D,S))}_{\phi(S)} = cS + p\int_S^\infty (x-S)f(x)\,dx + h\int_0^S (S-x)f(x)\,dx$$

$$\phi'(S) = c - p\int_S^\infty f(x)\,dx + h\int_0^S f(x)\,dx$$

$C(D,S) = cS + p(D-S)^+ + h(S-D)^+$

$$\phi'(S) = c - p\int_S^\infty f(x)\,dx + h\int_0^S f(x)\,dx$$

$$= c - p(1 - F(S)) + hF(S)$$

$$= c - p + (p+h)\,F(S).$$

Optimal S is solution to $F(S) = \dfrac{p-c}{p+h}$

$$\phi''(S) = (p+h)\,f(S) > 0$$