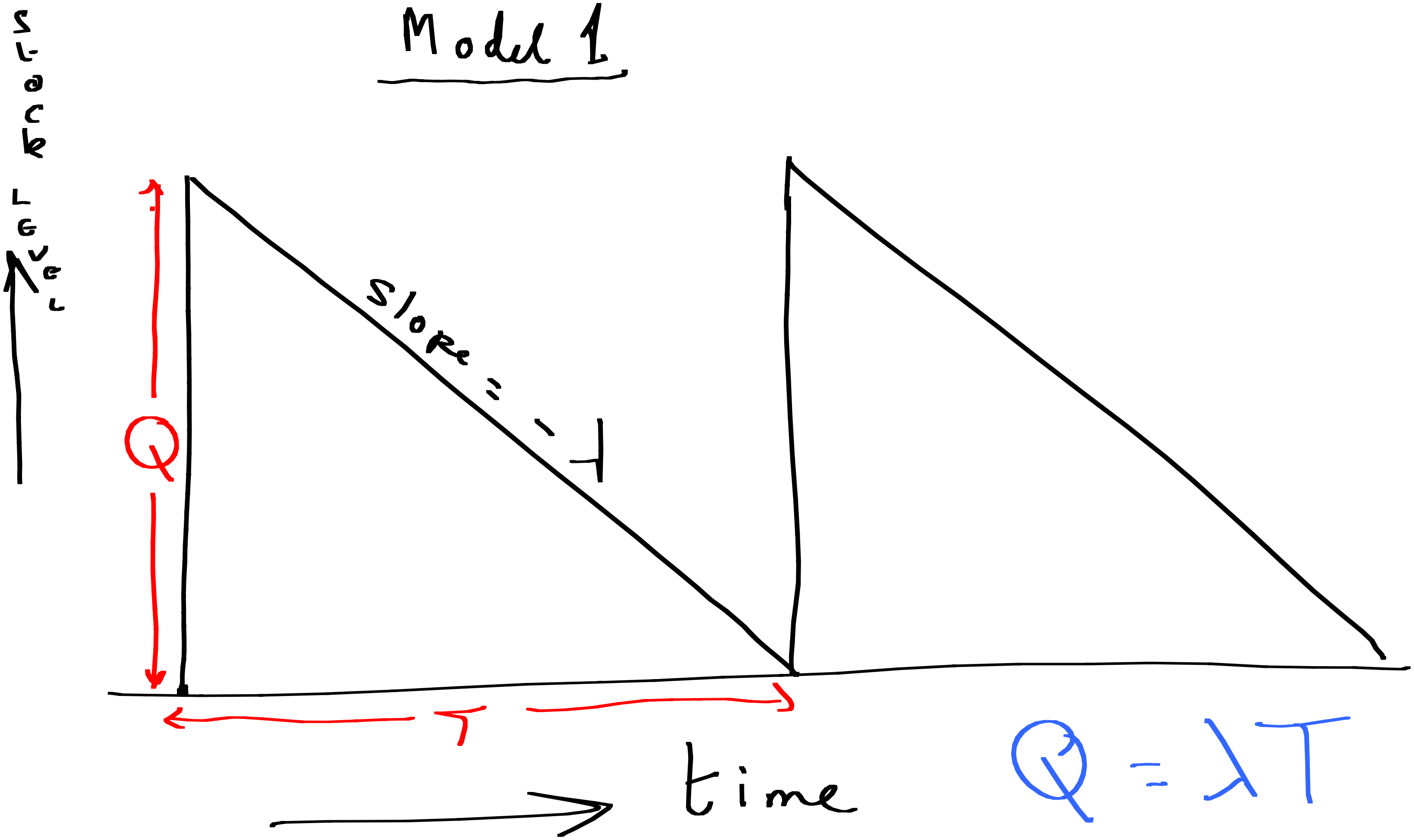
10\3\08

Inventory Control

Stock Levels.

EOQ : Economic Order Quantity

Model 1

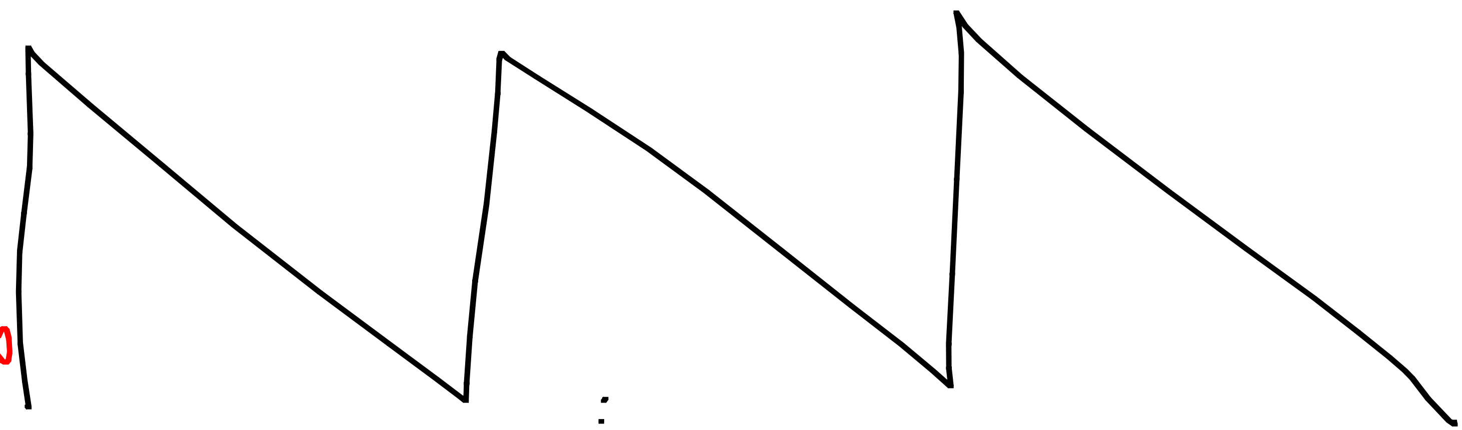


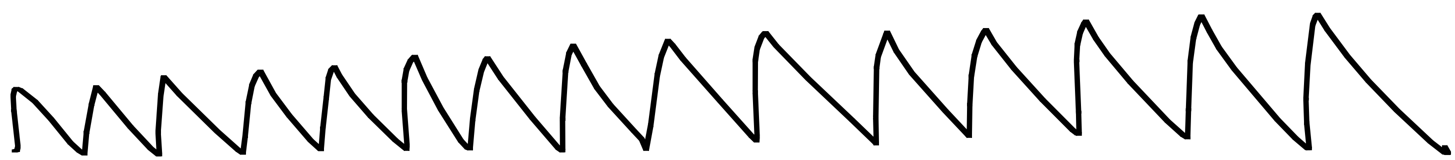
$$Q = \lambda T$$

Question: What is optimal value of Q.

Minimise Cost :

Choose between





A = cost of making an order

I = inventory cost per unit of time per item.

Minimise cost per unit time

Ordering Cost + Inventory Cost

$\frac{1}{T}$ = average number of orders per period

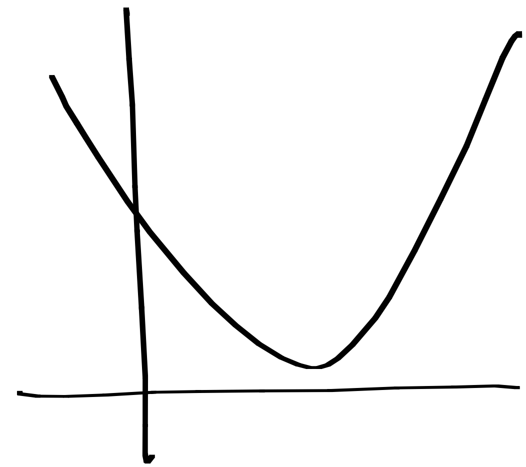
average inventory = $Q/2$

$$\frac{A}{T} + \frac{IQ}{2}$$

$$= \frac{A\lambda}{Q} + \frac{IQ}{2}$$

Choose Q to minimise

$$f(Q) = \frac{A\lambda}{Q} + \frac{IQ}{2}$$



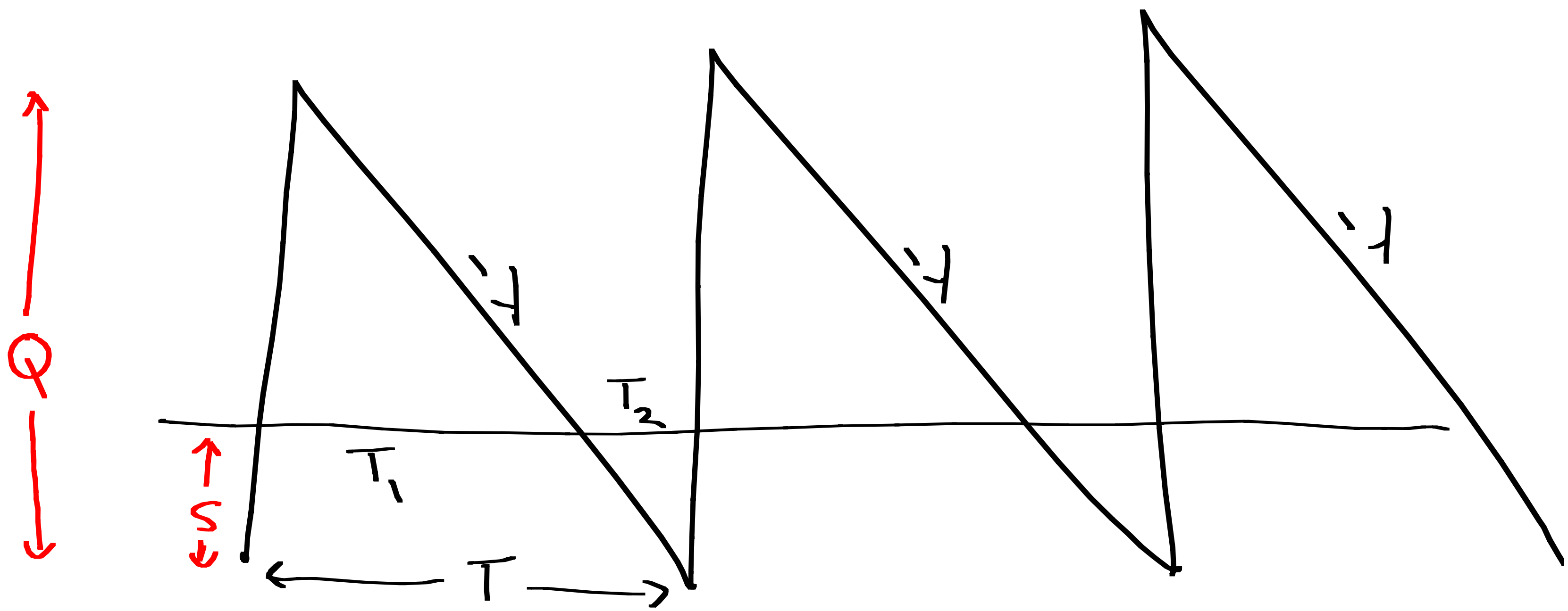
$$f'(Q) = -\frac{A\lambda}{Q^2} + \frac{I}{2}$$

$$Q^* = \sqrt{2A\lambda / I}$$ Wilson Lot Size Formula

$$T^* = \sqrt{2A / \lambda I}$$

$$\text{Min. Cost} = \sqrt{2A\lambda I}$$

Model 2



A = cost of making order

I = inventory cost

Π = penalty for being out of stock.

Ordering Cost + Inventory Cost + Penalty Cost

$$\frac{A}{T} + \frac{I(Q-S)T_1}{2T} + \frac{\pi S T_2}{2T}$$

$$Q = \lambda T \; ; \quad T_1 + T_2 = T \; ; \quad S = \lambda T_2$$

$$T = \frac{Q}{\lambda} \; ; \quad T_2 = \frac{S}{\lambda} \; ; \quad T_1 = \frac{Q}{\lambda} - \frac{S}{\lambda}$$

$$\underbrace{\frac{A\lambda}{Q} + \frac{I(Q-S)^2}{2Q} + \frac{\pi S^2}{2Q}}_{f(Q,S)}$$

Put $\frac{\partial f}{\partial Q} = \frac{\partial f}{\partial S} = 0$

$$\frac{\partial f}{\partial Q} = -\frac{A\lambda}{Q^2} + \frac{I}{2} - \frac{IS^2}{2Q^2} - \frac{\pi S^2}{2Q^2} = 0$$

$$\frac{\partial f}{\partial S} = \frac{I(S-Q)}{Q} + \frac{\pi S}{Q} = 0$$

Solve these equation

$$S = (2\lambda A I / \pi(\pi + I))^{1/2}$$

$$Q = Q_w((\pi + I)/\pi)^{1/2} \qquad Q_w = (2A\lambda/I)^{1/2}$$

$$K = K_w(\pi/(\pi + I))^{1/2} \qquad K_w = (2A\lambda I)^{1/2}$$