10/24/08

$$y_i = \min_j \left[c_{ij} + \alpha y_j \right] \qquad (*)$$

If $(*)$ holds then π is optimal.

Suppose $(*)$ does not hold.

$I = \{ i : (*) \text{ does not hold} \}$

If $i \in I$ let $\hat{\pi}(i)$ be index of minimum.

$$c_{i,\hat{\pi}(i)} + \alpha y_{\hat{\pi}(i)} = \min_j \left[c_{ij} + \alpha y_j \right]$$

$$\hat{\pi}(i) = \pi(i), \quad i \notin I.$$

Compare $\hat{\pi}$ & π

$$y_i \geq c_{i,\hat{\pi}(i)} + \alpha\, y_{\hat{\pi}(i)}$$

$$\hat{y}_i = c_{i,\hat{\pi}(i)} + \alpha\, \hat{y}_{\hat{\pi}(i)}$$

$\forall\ i \in I$

$$y_i - \hat{y}_i \geq \alpha \left[y_{\hat{\pi}(i)} - \hat{y}_{\hat{\pi}(i)} \right]$$

$$\vdots$$

$$\geq \alpha^k \left[y_{\hat{\pi}^k(i)} - \hat{y}_{\hat{\pi}^k(i)} \right]$$

$$\longrightarrow 0$$

Example

i	1	2	3
π	3	1	1

$$\begin{array}{c c} & j \\ i & \begin{bmatrix} 1 & 3 & 6 \\ 5 & 1 & 2 \\ 4 & 1 & 4 \end{bmatrix} \end{array}$$

$\alpha = 1/2$

Compute y:

$$y_1 = 6 + \tfrac{1}{2} y_3$$
$$y_2 = 5 + \tfrac{1}{2} y_1$$
$$y_3 = 4 + \tfrac{1}{2} y_1$$
$$= 4 + \tfrac{1}{2}\left(6 + \tfrac{1}{2} y_3\right)$$

$y_3 = \frac{28}{3}$ $\quad$ $y_1 = \frac{32}{3}$

$y_2 = \frac{31}{3}$

Check Optimality:

$$\begin{array}{cc} & j \\ i & \begin{bmatrix} 1 & 3 & 6 \\ 5 & 1 & 2 \\ 4 & 1 & 4 \end{bmatrix} \end{array}$$

$y_1 = \frac{32}{3}$

$y_3 = \frac{28}{3}$

$y_2 = \frac{31}{3}$

$i = 1$:

$\underline{1} + \frac{1}{2} \times \frac{32}{3}$ *

$3 + \frac{1}{2} \times \frac{31}{3}$

$6 + \frac{1}{2} \times \frac{28}{3}$

$i = 2$:

$5 + \frac{1}{2} \times \frac{32}{3}$

$1 + \frac{1}{2} \times \frac{31}{3}$ *

$2 + \frac{1}{2} \times \frac{28}{3}$

$i = 3$

$4 + \frac{1}{2} \times \frac{32}{3}$

$1 + \frac{1}{2} \times \frac{31}{3}$ *

$4 + \frac{1}{2} \times \frac{28}{3}$

$\hat{\pi} = (1, 2, 2)$

$$\begin{bmatrix} 1 & 3 & 6 \\ 5 & 1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$$

Compute y

$$y_1 = 1 + \frac{1}{2} y_1 = 2$$

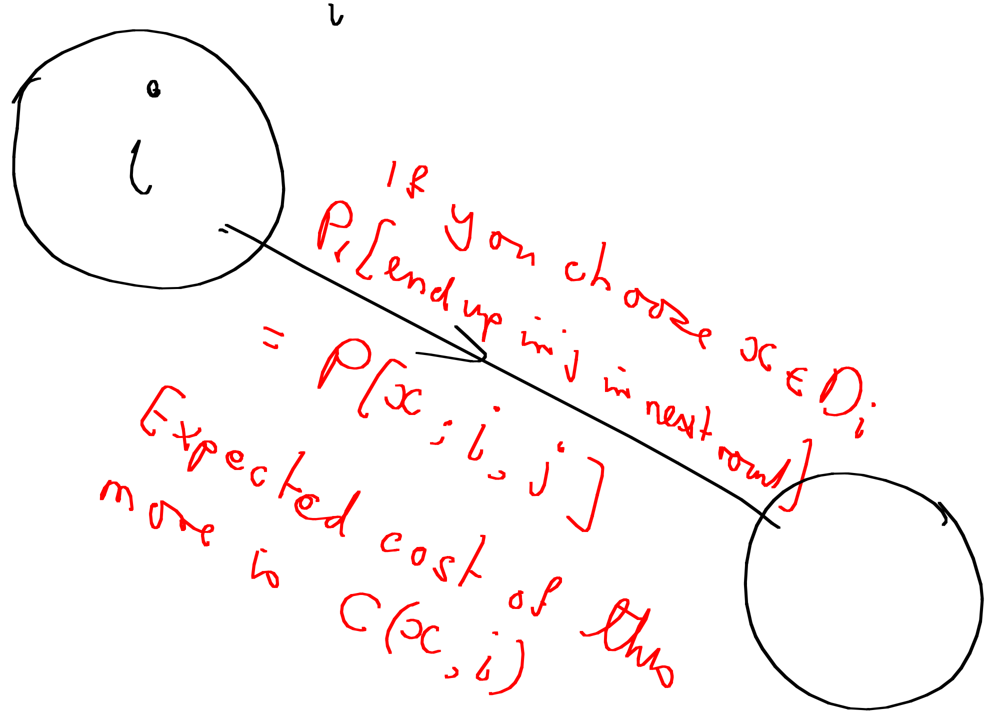
$$y_2 = 1 + \frac{1}{2} y_2 = 2$$

$$y_3 = 1 + \frac{1}{2} y_2 = 2$$

Check Optimality : Obvious

Add probability

$D_i = \{ \text{decisions available} \}$



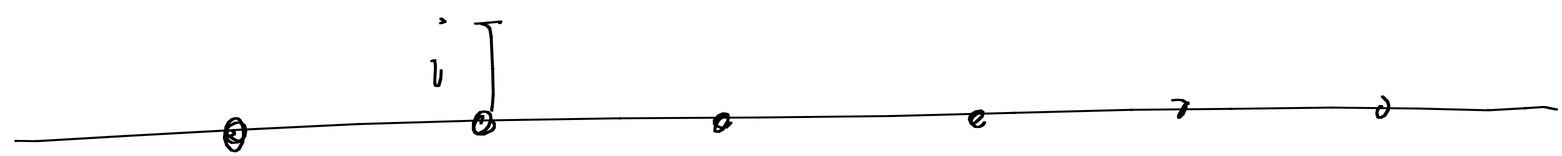
If you choose $x \in D_i$

$P_i[\text{end up in } j \text{ in next round}]$

$= P[x; i, j]$

Expected cost of this move is $C(x, i)$

Inventory problem



i = amount in stock

x = production level

d = demand , $P_d = \Pr[\text{demand} = d]$

$$P[x; i, j] = P_{x+i-j}$$

Policy π : $\pi(i) \in D_i$

$i \in V$.

Problem: choose π to minimise expected discounted cost of going indefinitely.