

10/22/08

n states $V = \{ \text{states} \}$



α = discount factor.

Policy: $\pi: V \rightarrow V$

When in i , go next to $\pi(i)$,
indefinitely.

y_v = discounted cost
 starting at i

$$= C_i, \pi(i) + \alpha y_{\pi(i)}$$

Solve these equations to get the y_i

$$A_{-v} = \begin{bmatrix} 1 & 1 & 1 & & \\ & 1 & 1 & & \\ & & 1 & & \\ & & & 1 & -\alpha \\ & & & & 1 \end{bmatrix}$$

NOW-SINGULAR

Want to find optimal $\pi \equiv \pi^*, y^*$

$$y_i^* \leq y_i \quad \forall i, \forall \pi$$

$y_{i,n}^*$ = min cost, starting at i ,
making n moves

$$= \min_j \{ c_{ij} + \alpha y_{j,n-1}^* \}$$

Let $n \rightarrow \infty$

$$y_i^* = \min_j \{ c_{ij} + \alpha y_j^* \} \quad \textcircled{**}$$

T^* is optimal $\Leftrightarrow$ $(**)$ holds.

(i) Suppose

$$y_i^* = \min_j [C_{ij} + \alpha y_j^*] \quad (**)$$

Let π be any pairing with corresponding y .

$$y_i = C_{i, \pi(i)} + \alpha y_{\pi(i)}$$

$$y_i^* \leq C_{i, \pi(i)} + \alpha y_{\pi(i)}^*$$

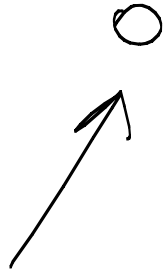
$$\frac{y_i - y_i^*}{\alpha} \geq \alpha [y_{\pi(i)} - y_{\pi(i)}^*]$$

$$Z_i \geq \alpha Z_{i, \pi(i)}$$

$$> \alpha^2 Z_{i, \pi^2(i)}$$

...

$$> \alpha^k Z_{i, \pi^k(i)}$$



$$\in Z_v \geq 0$$