10\15\08

First Passage time

$$f_{ij}^{(n)} = \Pr[\text{first visit to } j \text{ from } i \text{ occurs at step } n]$$

$$f_{ij}^{(1)} = P_{ij}$$

$$f_{ij}^{(n)} = \sum_{k \neq j} P_{ik} f_{kj}^{(n-1)}$$

$n \geq 2$

$$\mu_{ij} = E(\text{time of first visit to } j \text{ after leaving } i)$$

$$\mu_{ii} = \frac{1}{\pi_i}$$

$$\mu_{ij} = \sum_{n=1}^{\infty} n f_{ij}^{(n)}$$

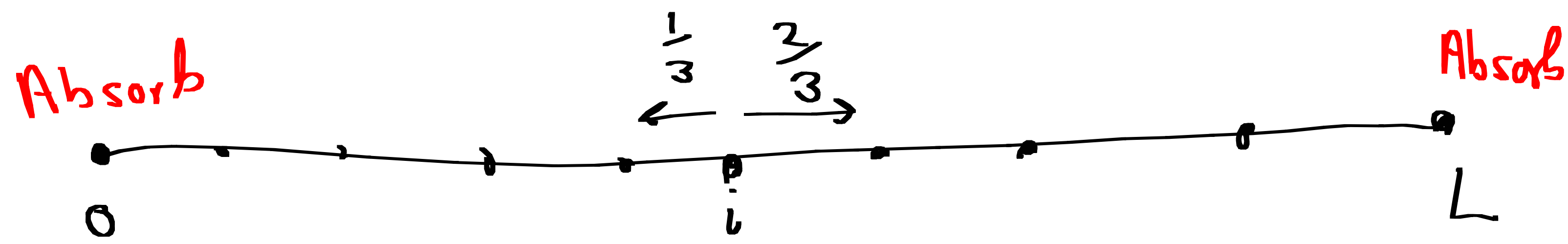
$$= 1 + \sum_{k \neq j} P_{ik} \mu_{kj}$$

Absorption Probabilities

Suppose there are S absorbing states S

$$f_{i,s} = \Pr[\text{absorption into } s \in S \text{ if started at } i]$$

$$= \sum_j p_{ij} f_{js}$$



$P_i = P_r[\text{absorbed at } 0, \text{ if started at } i]$

$P_0 = 1;\ P_L = 0;$

$0 < i < L: \quad P_i = \frac{1}{3} P_{i-1} + \frac{2}{3} P_{i+1} \qquad (1)$

The solution to (1) and $P_0 = 1$ has the form

$$P_i = A + \frac{B}{2^i} \quad \text{for } 0 \leq i < L$$

where $1 = P_0 = A + B$.

Also, $P_{L-1} = \frac{1}{3} P_{L-2} \Rightarrow A + \frac{B}{2^{L-1}} = \frac{1}{3}\left(A + \frac{B}{2^{L-2}}\right)$

$$\boxed{A = -\frac{1}{2^L - 1} \qquad B = \frac{2^L}{2^L - 1}}$$