Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 4: Due Monday October 29.

Q1
Find an optimal inventory policy for the model with the following parameters: It is a generalisation of Models 2 and 3 of notes.

A	Cost of making an order.
λ	Demand per period for items.
ψ	Arrival rate of ordered items.
I	Inventory cost per item per period.
π	Penalty cost per item out of stock per period.

1. First draw a diagram showing the inventory level over time and various parameters.
2. Then identify the various costs per period.
3. Optimize total cost.

Q2
Describe a modification of Dijkstra's algorithm that can be used to solve the following problem: Prove that your algorithm finds optimum paths.
(The standard proof for Dijkstra will work, with only very minor changes.)
You are given a digraph D with a distinguished vertex s. If $P = (s = x_0, x_1, \ldots, x_k)$ is a path and $P_i = (x_0, x_1, \ldots, x_i)$ for $i = 0, 1, \ldots, k$ then we define its *weight* $w(P)$ by

$$w(P_0) = 0 \textit{ and } w(P_i) = w(P_{i-1}) + \phi_{e_i}(w(P_{i-1}))$$

where for every edge e, ϕ_e is a non-negative monotone increasing function. (What this is saying is that the time to cross edge $e = (x, y)$ is a function of the time of arrival at x and increases with this time.)

Q3
How would you solve the following *bottleneck* assignment problem: You are given an $m \times m$ matrix of costs A and you are asked to find an assignment a which minimises

$$\max_{i=1,\ldots,m} A(i, a(i)).$$