Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 2: Due Monday September 24.

Q1 Consider an electronic system consisting of four components each of which must work for the system to function. The reliability of the system can be improved by installing several parallel units in one or more of the components. The following table gives the probability that the respective components will function if they consist of one, two or three parallel units.

Number of units	Component 1	Component 2	Component 3	Component 4
1	0.4	0.6	0.7	0.5
2	0.6	0.7	0.8	0.7
3	0.8	0.8	0.9	0.9

The probability that the system will function is the product of the probabilities that the respective components will function. The cost (in hundred's of dollars) of installing one, two or three parallel units in the respective compoonents is given by the following table.

Number of units	Component 1	Component 2	Component 3	Component 4
1	1	2	1	1
2	2	4	3	3
3	3	5	4	4

Because of budget limitations, a maximum of $1000 can be spent. use Dynamic Programming to determine how many parallel units should be installed in each of the four components in order to maximise the probability that the system will function.

Q2 A system can be in 3 states 1,2,3 and the cost of moving from state i to state j in one period is $c(i,j)$, where the $c(i,j)$ are given in the matrix below. The one period discount factor α is 1/2.

The aim is to find a policy which simultaneously minimises the discounted cost of operating from any starting state. Start with the policy

$$\pi(1) = 1, \pi(2) = 1, \pi(3) = 2.$$

Find an optimal policy.
The matrix of costs is

$$\begin{bmatrix} 8 & 2 & 3 \\ 5 & 2 & 8 \\ 1 & 7 & 2 \end{bmatrix}$$

Q3 You are an oil trader and the price of oil fluctuates in the following way: If the price per barrel in period i is p_i dollars then

$$p_{i+1} = \begin{cases} (1+\varepsilon)p_i & \textit{Probability } p \\ (1-\varepsilon)p_i & \textit{Probability } 1-p \end{cases}$$

The current price is one hundred dollars per barrel. Give a dynamic programming formulation to compute the maximum price per barrel one should pay now for the following option: Purchase oil at price c dollars per barrel, at any time $t = 1, 2, \ldots, n$.
Explanation: Here one is paying for the (hoped for) opportunity to purchase the oil cheaply in the future.