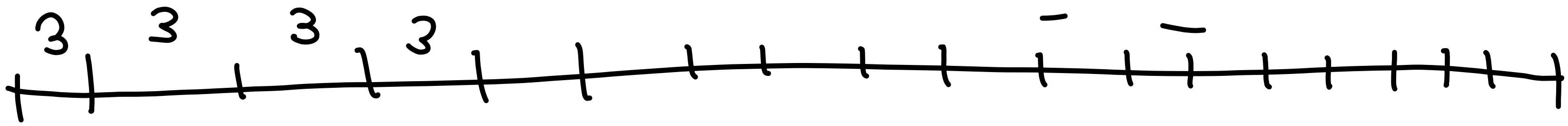
9\14\ 2007

demand



Sequence of costs

(1) 2 2 2 2 2 . . . . 2 . .

(2) 3 3 3 3 3 . . 3 . .

(3) 1 3 1 3 1

(1) is better than (2)

What about (1) & (3)

(1) has a better average cost.

Compare

2 2 2 2 2 2 2.

1 3 1 3 1 3 1 ...

Same average cost.

(a) Inflation. Suppose money decreases in value by a factor $\alpha < 1$ every year.

(i) In today's dollars, the cost is $2 + 2\alpha + 2\alpha^2 + 2\alpha^3 + \cdots$

(ii) $1 + 3\alpha + \alpha^2 + 3\alpha^3 + \cdots = \dfrac{1}{1-\alpha^2} + \dfrac{3\alpha}{1-\alpha^2} < \dfrac{2}{1-\alpha}$

(b) Suppose you have the possibility of earning $1+r$ interest per year (indefinitely)

How much need you invest now to pay for the whole thing

$$2 + \frac{2}{1+r} + \frac{2}{(1+r)^2} + \cdots$$

$$\alpha = \frac{1}{1+r}$$

or again

$$1 + \frac{3}{1+r} + \frac{1}{(1+r)^3} + \cdots$$

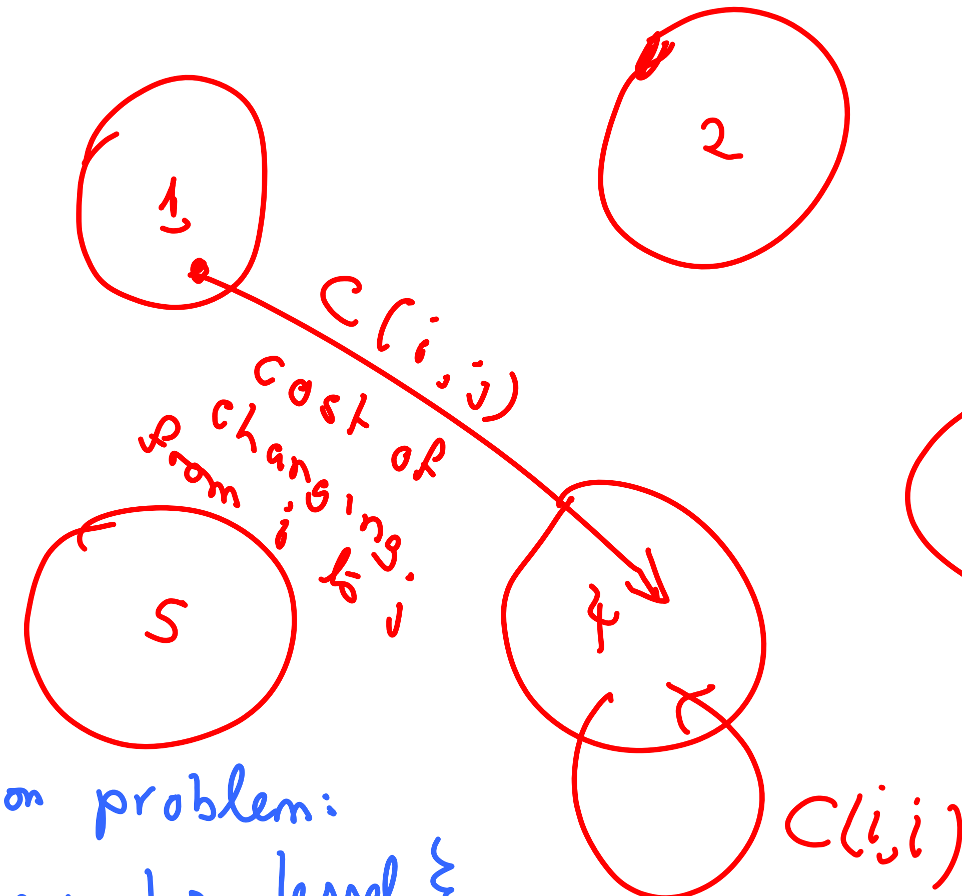
$$a_0 + a_1\alpha + a_2\alpha^2 + \alpha_3\alpha^3, \ldots$$

is the NET PRESENT WORTH of DISCOUNTED COST

$a_0, a_1, a_2, \ldots$

We try to optimise

Assume then that there is a set V of possible states of a "system".



α = discount factor.

$V = \{1, 2, 3, 4, 5\}$

Production problem:

$V = \{$inventory levels$\}$

From a sequence of states

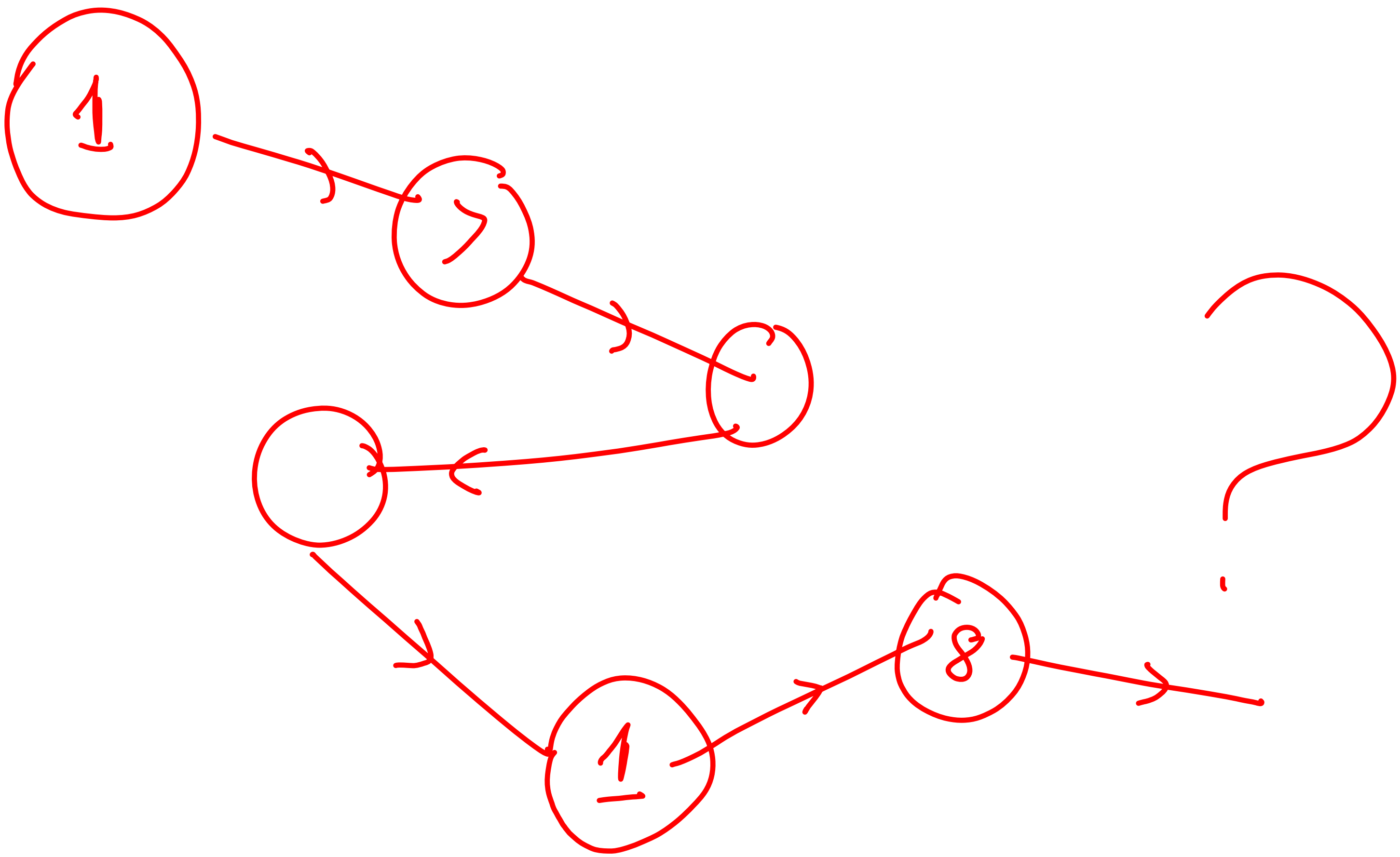
$i_1, i_2, i_3, i_4, \ldots$

we get a "cost"

$$C(i_1, i_2) + \alpha C(i_2, i_3) + \alpha^2 C(i_3, i_4) + \cdots$$

This is what we want to minimize.

Optimal strategy



Claim

For each state i, there is an optimal choice $\pi^*(i)$ for the next state to go to.

Now my problem is to find the "optimal" $\pi : V \to V$

choices is $|V|^{|V|}$

Solution Strategy:

Choose an initial $\pi \in \overbrace{V^V}^{\{\text{policies}\}}$

e.g. $V = \{1, 2, 3\}$

i	1	2	3
$\pi(i)$	1	2	2

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & 3 & 5 \\ 4 & 4 & -10 \end{bmatrix}$$

Check for optimality

If not optimal, improve current one