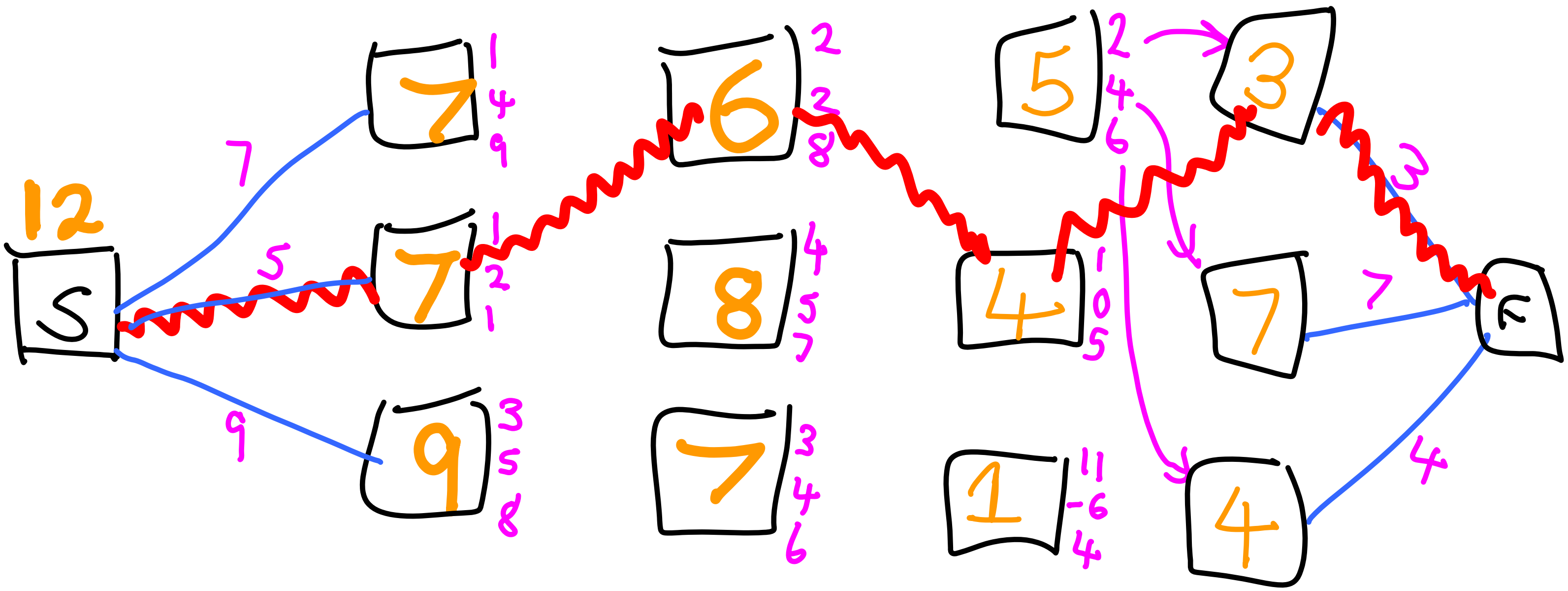
8/29/2007

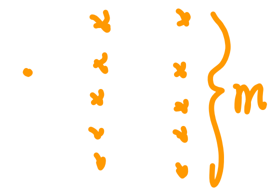
minimum distance
$f_r(i)$



$f_r[i]$ = minimum length of a path from i^{th} pass in range r to F.

$$= \min_j \left[c_r(i,j) + f_{r+1}(j) \right]$$

How long does it take to run the algorithm.



Assume each range has m passes

* $$f_r(i) = \min_j \left[c_r(i,j) + f_{r+1}(j) \right]$$

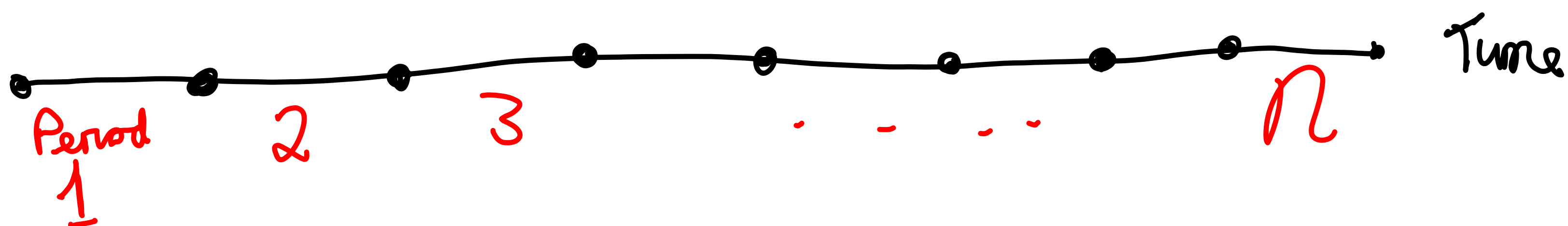
mn numbers to compute

takes "order" m time

Total "work" $\approx m^2 n$: #paths $= m^n$ much bigger.

Simple Production Problem

Meet demand for some item (cars) for next n periods.



(i) Demand is given $d_1, d_2, \dots, d_n \geq 0$.

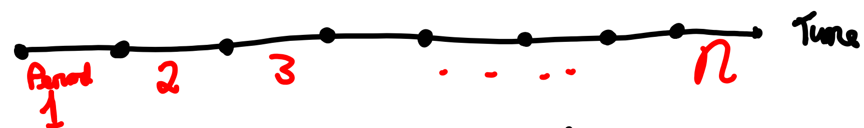
(ii) $c(x)$ = cost of making x units in a period.

(iii) Must meet demand.

(iv) Can only store $\leq H$ units.

Simple Production Problem

Meet demand for some item (cars) for next n periods.



Period ≡ range

stock level ≡ pass

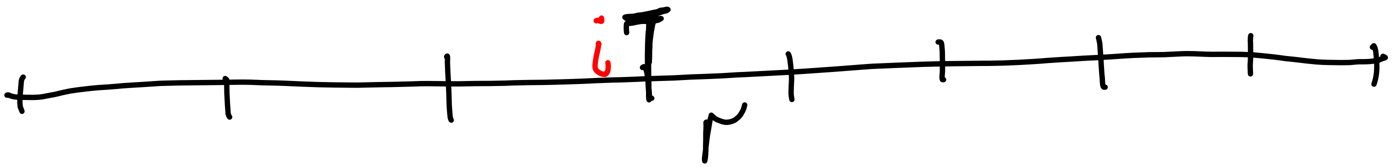
(i) Demand is given $d_1, d_2, \ldots, d_n \geq 0$.
(ii) $c(x)$ = cost of making x units in a period.
(iii) Must meet demand.
(iv) Can only store $\leq H$ units.

This is an n period problem.

(i) Make a decision – how much to make in period 1.
Left with a $n-1$ period problem.

[One period i ⊢ d ⊣ make $d-i$]

Decision depends on amount in stock at start of period.



$f_r(i)$ = minimum cost of meeting demand in $r, r+1, \dots n$ starting with i in stock.

$$f_r(i) = \min_{\substack{x \geq 0 \\ i+x \geq d_r \\ i+x \leq H+d_r}} \left[c(x) + f_{r+1}(i+x-d_r) \right]$$

$$f_{n+1}(i) = 0$$

Throw away any excess production from period $n+1$!

Mountain Range

$i + x - d_r = j$

$$\begin{tikzcd} \boxed{i} \arrow[dr, "C(j+d_r-i)"] & \\ & \boxed{j} \\ r & r+1 \end{tikzcd}$$

H+1 passes in each range

Example: $H = 3$; $d_j = 4$; $c(x) = 20x - x^2$

$n = 4$ $\qquad x \le 20$

Period 4

best choice of production.

i	$f_4(i)$	$x_4(i)$	$f_3(i)$	$x_3(i)$	$f_2(i)$	$x_2(i)$	$f_1(i)$	$x_1(i)$
0	64	4	110	7				
1	51	3						
2	36	2						
3	19	1						

$r = 3$, $i = 0$:

$x = 4$	$64 + 64$
$x = 5$	$75 + 51$
$x = 6$	$84 + 36$
$x = 7$	$91 + 19$ *