10/5/2007

Maximize $\xi$

Dual Var: $q_j$

$$\xi - \sum_{i=1}^{m} a_{ij} P_i \leq 0, \quad j = 1, 2, \cdots, n$$

$$P_1 + \cdots + P_m = 1$$

$$P_1, P_2, \cdots P_m \geq 0$$

minimize $\eta$

$$\eta \geq \sum_{j=1}^{n} a_{ij} q_j \quad i = 1, 2, \cdots, m$$

$$q_1 + \cdots + q_n = 1$$

$$q_1 \cdots q_n \geq 0$$

$\eta$

Maximize

$$c_1 x_1 + c_2 x_2 + c_3 x_3$$

Dual

$y_1$: $a_{11} x_1 + a_{12} x_2 + a_{13} x_3 \leq b_1$

$y_2$: $a_{21} x_1 + a_{22} x_2 + a_{23} x_3 \geq b_2$

$y_3$: $a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3$

$$x_1 \geq 0 \quad x_2 \leq 0$$

$\Rightarrow$

Minimize

$$b_1 y_1 + b_2 y_2 + b_3 y_3$$

$$a_{11} y_1 + a_{21} y_2 + a_{31} y_3 \geq c_1$$

$$a_{12} y_1 + a_{22} y_2 + a_{32} y_3 \leq c_2$$

$$a_{13} y_1 + a_{23} y_2 + a_{33} y_3 = c_3$$

$$y_1 \geq 0 \quad y_2 \leq 0$$

minimize $\eta$

$$q_1 + \cdots + q_n = 1$$

$$\eta - a_{ij} q_j \geq 0$$

$$q_j \geq 0$$

Contraint $\xi$

Constraint $P_i$

# Dominant Strategies

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 6 & 5 & 3 & 4 \\ \cancel{2} & \cancel{2} & \cancel{2} & \cancel{3} \\ \cancel{1} & \cancel{2} & \cancel{2} & \cancel{3} \end{bmatrix} \end{array}$$

Row 1 dominates Row 2

Col 3 dominates rest

# Simple Games

(i) Symmetric Game

$$A^T = -A \qquad n \times n \text{ game}$$

$$\begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & 3 \\ 1 & -3 & 0 \end{bmatrix}$$

$$\sum_i \sum_j a_{ij} p_i p_j = 0$$

$$a_{ij} p_i p_j + a_{ji} p_j p_i = 0$$

$$\forall p : \text{PAY}(p, p) = 0$$

$$\text{PAY}[\quad]$$

$$P_A \leq 0$$

$$P_B \geq 0$$

# General Games

$n$ players.

Player $i$ has $m_i$ pure strategies

$A_k(i_1, i_2, \dots, i_n)$ = pay-off to player $k$

if player $t$ plays $i_t$, $t = 1, 2, \dots, n$

Mixed Strategies: player $i$ has a vector of probabilities.

$A_k(\underset{\sim}{p_1}, \underset{\sim}{p_2}, \dots, \underset{\sim}{p_n})$

$p_1^*, p_2^*, \ldots p_n^*$ is a Nash equilibrium if

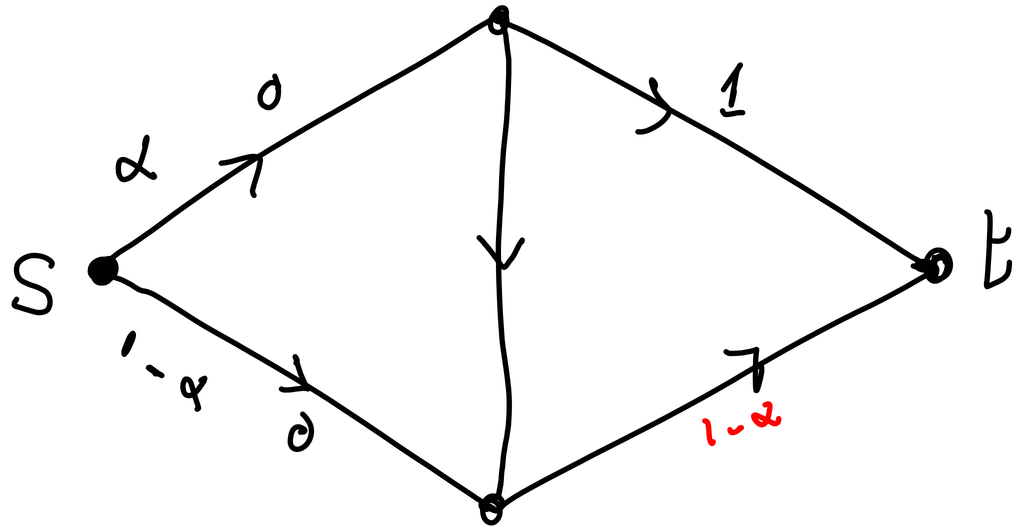
$$A_k(p_1^*, p_2^*, \ldots p_k^*, \ldots p_n^*)$$

$$\geq A_k(p_1^*, p_2^*, \ldots, p, \ldots p_n^*)$$

$\forall\, k \,\&\, p$

Thm: There is always at least one Nash equilibrium (of mixed strategies).

Hard to compute a Nash Equilibrium




$\underline{1 \text{ unit}}$ of "traffic flow" from $s \longrightarrow t$

$N = 10^{10}$ cars.

Nash Eq: Every-one along bottom cost $N$

With $\alpha$: $N\left[\alpha + (1-\alpha)^2\right]$ $\qquad \frac{3}{4}N$

$= N\left[1 - \alpha + \alpha^2\right] \leftarrow$ minimised at $\alpha = \frac{1}{2}$