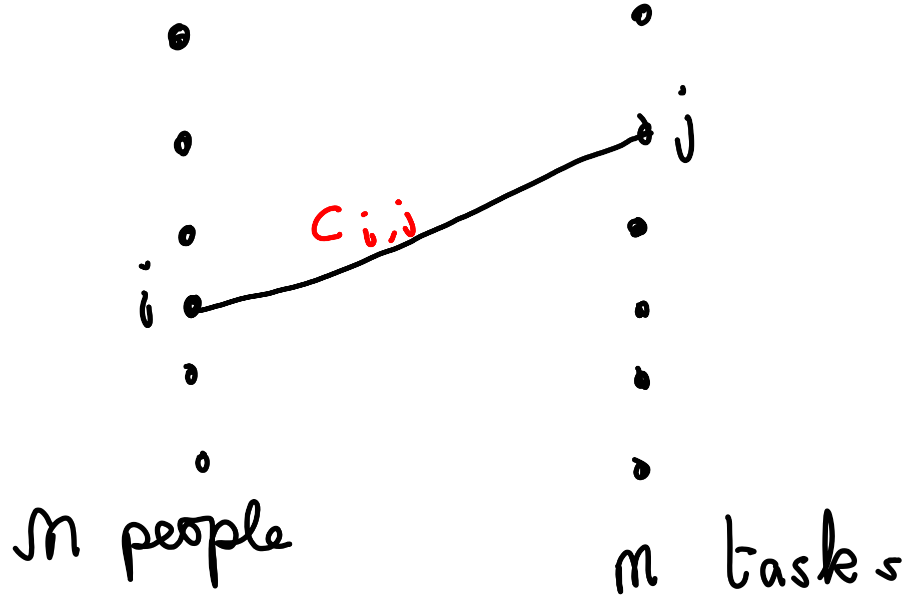
10\17\2007

Assignment Problem

$c_{i,j}$ = cost if person i does task j

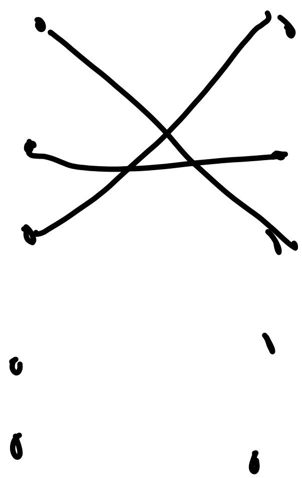


Person i does one task $a(i)$

Each task is done by one person

Problem: $\underset{a}{\text{minimise}} \sum_i c_{i,a(i)}$.

k - assignment : An assignment of first k people to first k tasks.



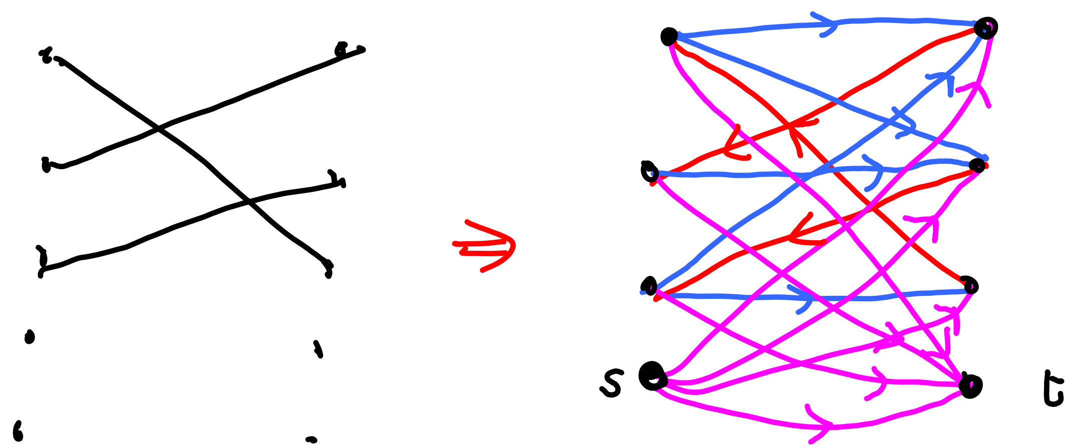
3 - assignment.

Idea : Given minimum cost k-assignment, finding minimum cost (k+1) - assignment is a shortest path problem.

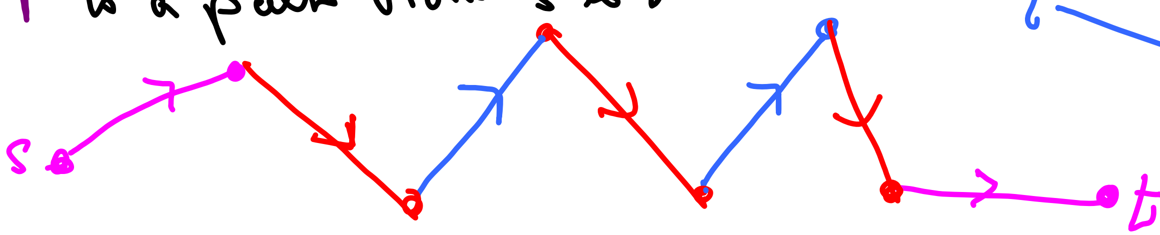
Finding min. cost 1- assignment is trivial

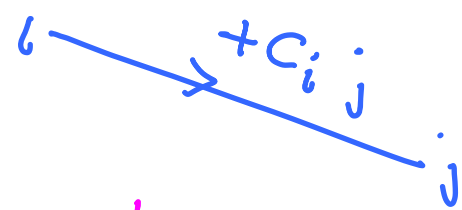
Suppose we have minimum cost k-assignment.

$D[k, a]$

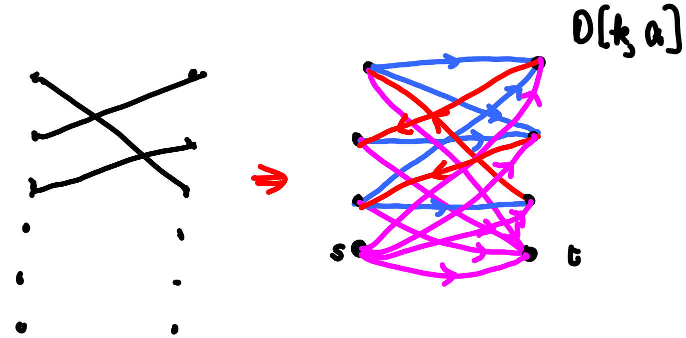


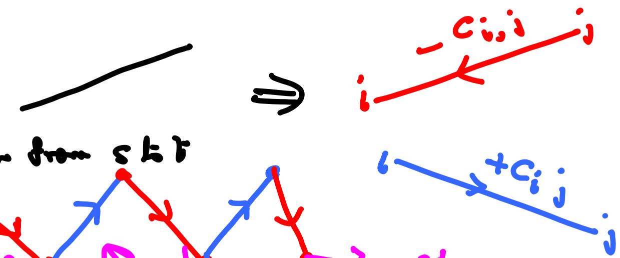
P is a path from s to t



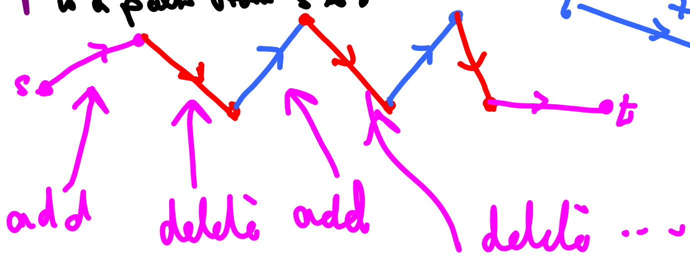


Suppose we have minimum cost k-assignment



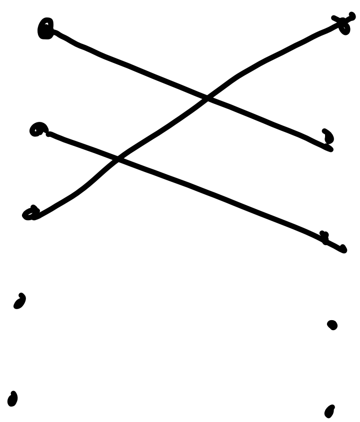


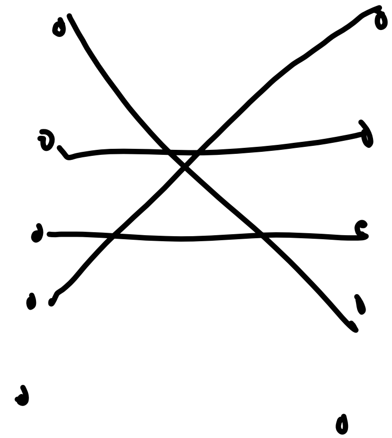
P is a path from s to t



If I use P, I get a $(k+1)$-assignment

$$\ell(P) = \left(\text{cost of added edges}\right) - \left(\text{cost of deleted edges}\right)$$





Min cost k-assignment

A (edges)

Any $k+1$-assignment

B (edges)

$A \oplus B$

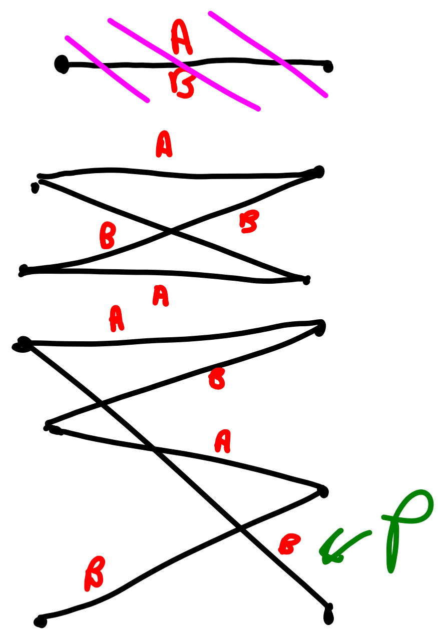
$(A - B) \cup (B - A)$

Since A is minimum cost k-assignment,

$l(\text{Cycle}) \geq 0$

neg. length

$\Rightarrow$ A can be improved.

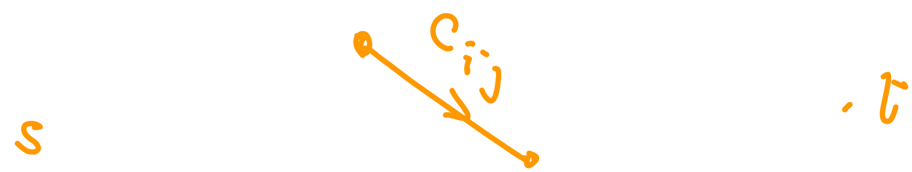


$$\text{Cost}(B) = \text{cost}(A) + l(P) + \sum l(\text{Cycles})$$

As described algorithm runs in $O(n^4)$ time.

Can be reduced to $O(n^3)$

[Basically one can modify arc lengths to be ≥ 0.



$$c_{ij} \longrightarrow c'_{ij} = c_{ij} - \lambda_i + \lambda_j$$

$$l'(P) = l(P) - \lambda_s + \lambda_t$$

Assigment Problem = Integer Program

$$\text{Minimise} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \, x_{ij}$$

s.t.

$$\sum_{i=1}^{m} x_{ij} = 1 \qquad \forall j$$

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad \forall i$$

$$x_{ij} = 0 \text{ or } 1$$

Can be replaced by $0 \le x_{ij} \le 1$