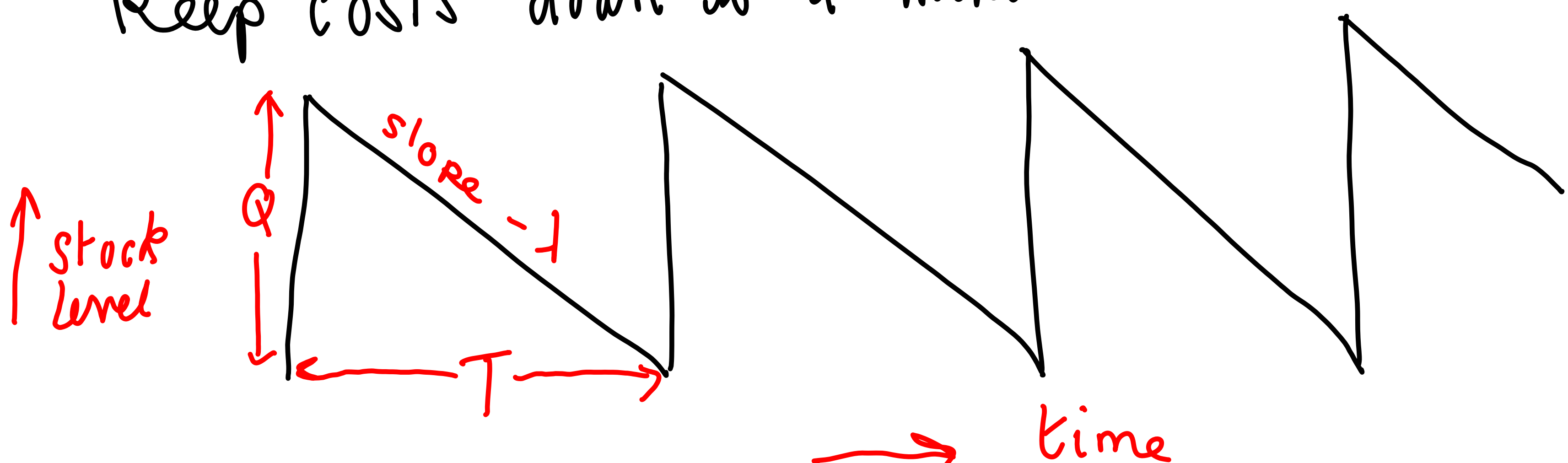
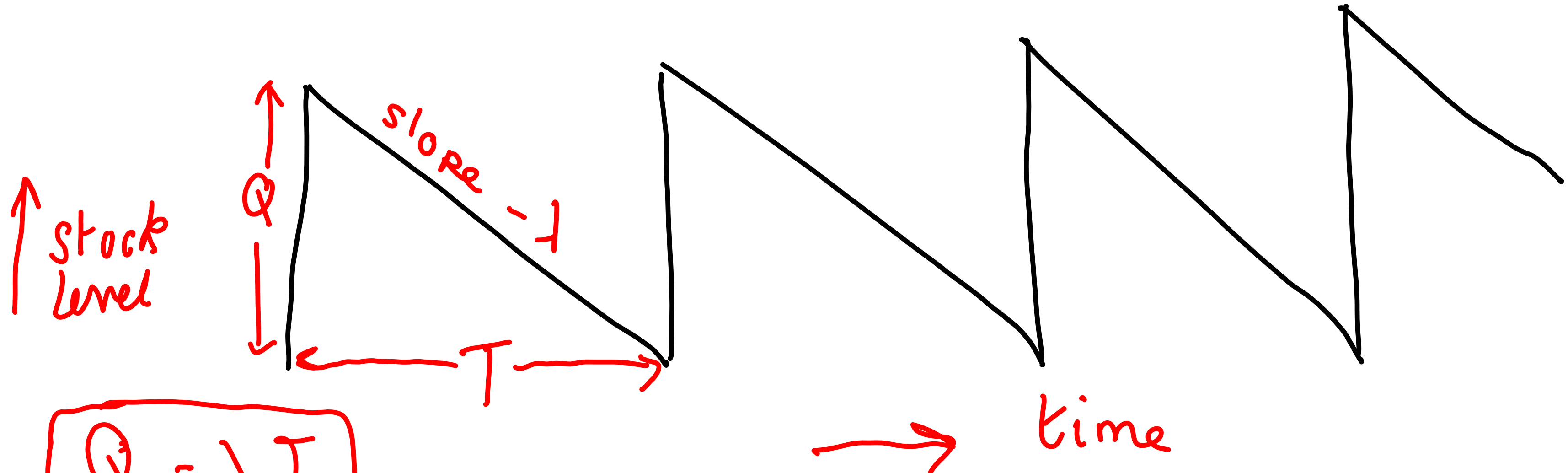
10\08\2007

Model 1

Demand for an item is λ per period.
At constant time intervals T, you buy a quantity Q.
Keep costs down to a minimum.





$$\boxed{Q = \lambda T}$$

Costs:

(i) Purchasing cost of items —
In time t, — λt independent of Q, T

(ii) Order cost: A every time you make an order

(iii) inventory cost: I per unit, per period.

$$\text{Average Cost} = \text{Order Cost per period} + \text{average inventory cost}$$

(i) Order Cost: $\dfrac{A}{T}$
per period

$1/T$ orders per period

(ii) Inventory cost: $\frac{1}{2} I Q$

Total Cost $= \dfrac{A}{T} + \dfrac{IQ}{2}$

$= \dfrac{A}{T} + \dfrac{I \lambda T}{2}$

Wilson's Lot Size Formula

$$Q^* = \sqrt{\frac{2A\lambda}{I}}$$

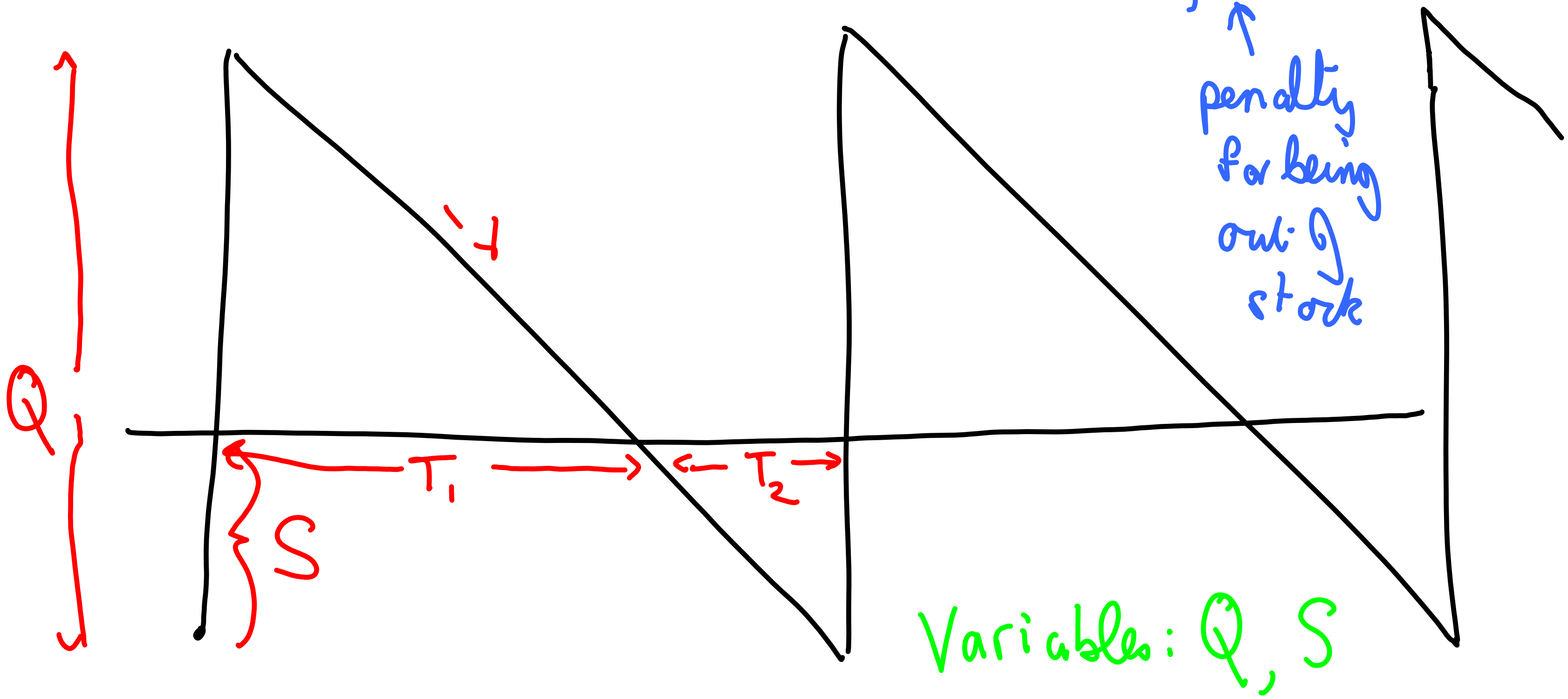
Minimise:

$$-\frac{A}{T^2} + \frac{I\lambda}{2} = 0$$

$$T^* = \sqrt{\frac{2A}{\lambda I}}$$

Costs: A, I, π

penalty for being out of stock



Variables: Q, S

Costs:

Ordering + Inventory + Penalty

$$\frac{A}{T} + \frac{I(Q-S)\,T_1}{2T} + \frac{\pi\, S\, T_2}{2T}$$

$$\Downarrow$$

$$\frac{A\lambda}{Q} + \frac{I(Q-S)^2}{2Q} + \frac{\pi S^2}{2Q}$$

$T_1 + T_2 = T$

$Q - S = \lambda T_1$

$S = \lambda T_2$

$$\underbrace{\text{Total Cost}}_{K} = \frac{A\lambda}{Q} + \frac{I(Q-S)^2}{2Q} + \frac{\pi S^2}{2Q}$$

$$\frac{\partial K}{\partial Q} = \frac{\partial K}{\partial S} = 0$$

$$S = \sqrt{\frac{2\lambda A I}{\pi(\pi + I)}} \qquad Q = \sqrt{\frac{2\lambda A}{I}} \cdot \sqrt{\frac{\pi + I}{\pi}}$$