10\1\2007

Two Person Zero Sum Games.

Game

Player B

$$\begin{array}{cc} & j \\ i & \begin{bmatrix} \text{matrix } A \\ x \end{bmatrix} \end{array}$$

Player A

$A[i, j] = x$

A = Row Player : Chooses i

B = Column Player : Chooses j

B pay x units to A.

$$\begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

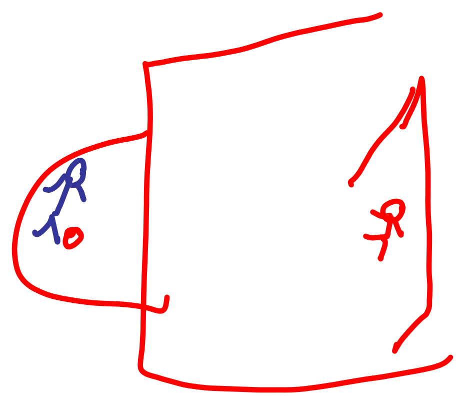
Tennis

A

B

Serving

Soccer.



Game is played over & over again

A tries to maximise average winnings

B tries to minimise A's average winnings

$S_A = \{$ of A's possible strategies $\}$

Sequence of functions

S_B

$(i) = i, i, i, i, i, \ldots$

Pure Strategy

If $u \in S_A$

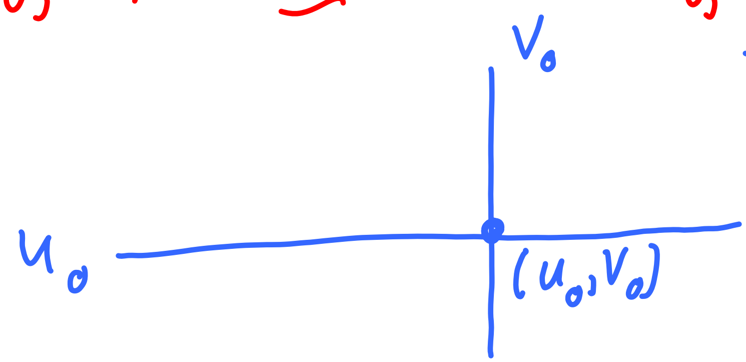
$v \in S_B$

then $PAY(u,v)$ = average payoff to A

Stable Solution:

(u_0, v_0) is stable if

$$PAY(u_0, v) \ge PAY(u_0, v_0) \ge PAY(u, v_0)$$



largest entry in column v_0

&

smallest entry in row u_0

$$\begin{array}{c|ccc} & P & R & S \\ \hline P & 0 & 1 & -1 \\ R & -1 & 0 & 1 \\ S & 1 & -1 & 0 \end{array}$$

Is there a stable solution using pure strategies?

Is there (u_0, v_0) which is a column max & a row min

Every row min -1

Every col max 1

What are A's guranteed winning

$$P_A = \max_{u} \text{ROWMIN}(u)$$

If A chooses u then in long run A will get ROWMIN(u)

$$\begin{array}{cc} \underline{u} & \begin{bmatrix} \\ * & * & * & * & * & * & * & * \\ \\ \end{bmatrix} \end{array}$$

B should lose no more than

$$P_B = \min_{v} \ \mathrm{COLMAX}(v)$$

$$\begin{array}{c} v \\ \left[\begin{array}{ccc} & \times & \\ & \times & \\ & \times & \\ & \times & \\ & \times & \\ & \times & \\ & \times & \\ & \times & \end{array}\right] \end{array}$$

<u>Claim:</u>

(i) $P_A \leq P_B$

(ii) $P_A = P_B$ iff $\exists$ stable solution

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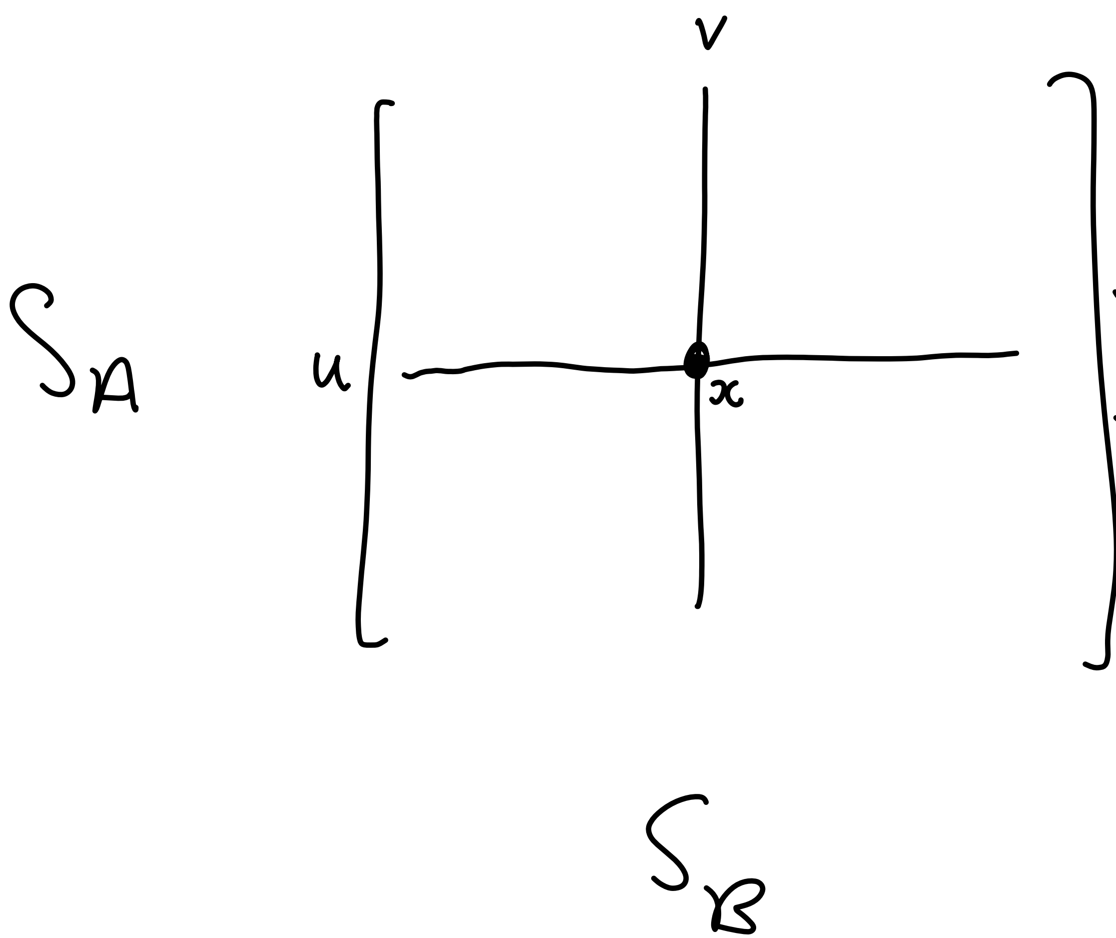

largest

$$\mathrm{ROWMIN}(u) \leq x \leq \mathrm{COLMAX}(v)$$

smallest