Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 2: Due Monday October 9.

Q1 Consider an electronic system consisting of four components each of which must work for the system to function. The reliability of the system can be improved by installing several parallel units in one or more of the components. The following table gives the probability that the respective components will function if they consist of one, two or three parallel units.

Number of units	Component 1	Component 2	Component 3	Component 4
1	0.4	0.6	0.7	0.5
2	0.6	0.7	0.8	0.7
3	0.8	0.8	0.9	0.9

The probability that the system will function is the product of the probabilities that the respective components will function. The cost (in hundred's of dollars) of installing one, two or three parallel units in the respective compoonents is given by the following table.

Number of units	Component 1	Component 2	Component 3	Component 4
1	1	2	1	1
2	2	4	3	3
3	3	5	4	4

Because of budget limitations, a maximum of \$1000 can be spent. use Dynamic Programming to determine how many parallel units should be installed in each of the four components in order to maximise the probability that the system will function.

Q2 A system can be in 3 states 1,2,3 and the cost of moving from state i to state j in one period is $c(i,j)$, where the $c(i,j)$ are given in the matrix below. The one period discount factor α is 1/2.

The aim is to find a policy which simultaneously minimises the discounted cost of operating from any starting state. Start with the policy

$$\pi(1) = 1, \pi(2) = 1, \pi(3) = 2.$$

Find an optimal policy.
The matrix of costs is

$$\begin{bmatrix} 7 & 2 & 3 \\ 5 & 2 & 7 \\ 1 & 6 & 2 \end{bmatrix}$$

Q3 A mathematics student has been asked to compute the product $A_1 \times A_2 \times \cdots \times A_n$ where each A_i is an $m_i \times m_{i+1}$ matrix. It will cost the student $\$pqr$ to multiply a $p \times q$ matrix by a $q \times r$ matrix. Describe a dynamic programming algorithm for finding the cheapest way of multiplying the n matrices together.
To see that there is a problem: let $\underline{m} = (10, 5, 8, 12)$. There are two ways to compute $A_1A_2A_3$: compute A_1A_2 first and then multiply this by A_3 – total cost 400+960=1360. The other way is to compute A_2A_3 first and then multiply this by A_1 – total cost 480+600=1080.