Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 2.

Q1 Solve the following 2-person zero-sum games:

$$\begin{bmatrix} 6 & 2 & 4 \\ 5 & 2 & 5 \\ 4 & 1 & -3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 & 1 & 0 & -1 \\ 4 & 3 & 2 & 1 & -1 \\ 1 & 1 & 0 & -1 & 1 \\ 2 & 1 & 1 & -2 & -2 \\ 4 & 1 & 0 & -2 & -3 \end{bmatrix}$$

Q2 Suppose the $n \times n$ matrix A is such that all row and column sums are equal to the same value C. What is the solution to this game?

Q3 Formulate the following problems as integer programs:

(a) The government has asked for and received bids on m construction projects from each of n firms. No firm will be awarded more than one contract and for political reasons no more than p large contracts are to go to foreign firms. Projects $1, 2, \ldots, \ell$ are large and firms $1, 2, \ldots, f$ are foreign. If $b_{i,j}$ is the bid by firm i for project j, which bids should be accepted to minimise the total cost?

(b) For the purpose of fire safety, a town is divide into n areas. The council has decided to build p fire stations. m possible sites have been found. Let $t_{i,j}$ denote the time taken to drive from area i to area j. The cost of locating a fire station at location i is f_i. Each area must be within driving time τ of a fire station. Where should the fire stations be located to minimise cost? [**Hint:** Its a set-covering problem.]

(c) An assembly line consists of a sequence of locations called work stations. The manufacture of a certain object requires m separate jobs to be undertaken with job i requiring t_i minutes. The jobs are to be allocated to work stations so that each station completes a set of jobs and then passes the object onto the next station on the line and waits to receive the next object from the previous station on the line. The combined time of all jobs assigned to any station must not exceed T the cycle time. Also there are a number of precedence relations between jobs indicated by the digraph $D = (V, A)$

where $(i, j) \in A$ if job i must precede job j. The problem is to open as few work stations as possible consistent with the cycle time.

Q4 Solve the following problem by a cutting plane algorithm:

$$\begin{array}{lrcrcrl} \text{minimise} & 4x_1 & + & 5x_2 & +3x_3 & & \\ \text{subject to} & & & & & & \\ & 2x_1 & + & x_2 & -x_3 & \geq & 2 \\ & x_1 & + & 4x_2 & +x_3 & \geq & 13 \end{array}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer.}$$

Q5 Solve the following problem by a branch and bound algorithm:

$$\begin{array}{lrrrrl} \text{Maximise} & 4x_1 & -2x_2 & +7x_3 & -x_4 & \\ \text{subject to} & & & & & \\ & x_1 & & +5x_3 & & \leq 10 \\ & x_1 & +x_2 & -x_3 & & \leq 1 \\ & 6x_1 & -5x_2 & & & \leq 0 \\ & -x_1 & & +2x_3 & -2x_4 & \leq 3 \end{array}$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

$$x_1, x_2, x_3 \text{ integer.}$$