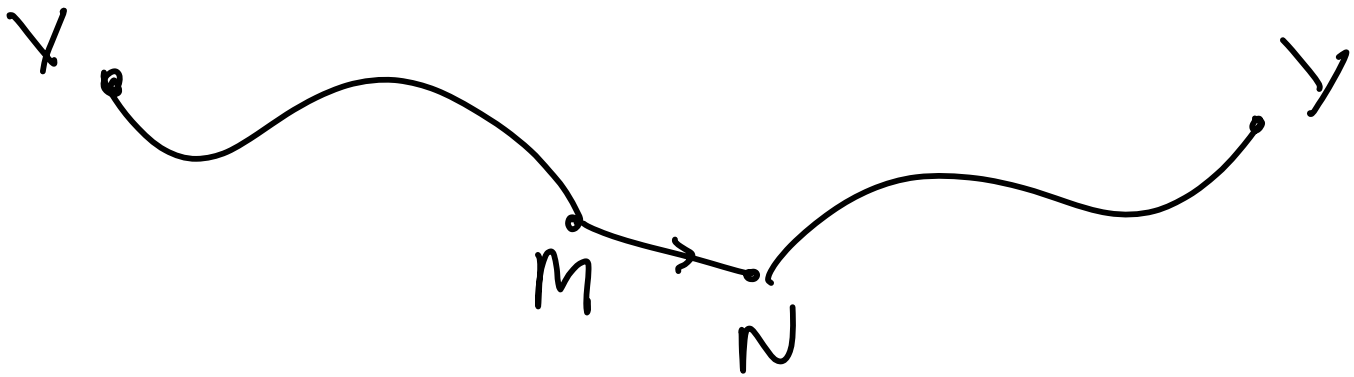
4/26/10



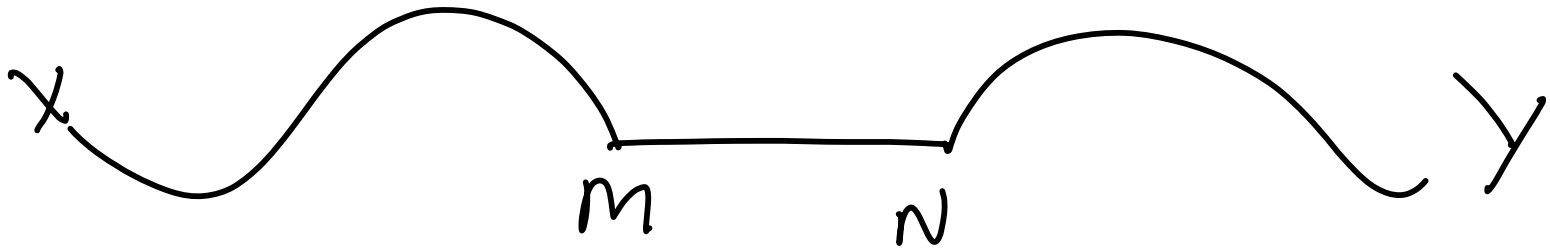
$\theta_{M,N}$ = # X, Y such that $(M,N) \in \gamma_{X,Y}$

Claim: $\boxed{\theta_{M,N} \leq |\gamma_{M}|}$

$$\rho \leq \frac{1}{\frac{1}{|m|} \times \frac{1}{|E|}} \times \frac{1}{|m|} \times \frac{1}{|m|} \times 2n \times \max_{M,N} \theta_{M,N}$$

$$= 2|E|n$$

$1 - \lambda_2 \geq \frac{1}{n^3}$



For M, N. How much of X, Y do we need to know, in order to fix X, Y.

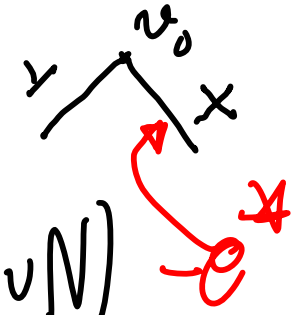
Intuition: $X \oplus Y = \underbrace{Z_1 \cup Z_2}_{} \subseteq \mathbb{M}$

$X = M \oplus Z_1$

$Y = N \oplus Z_2$

choices for X, Y = # choices for $(Z_1, Z_2) \leq |\mathbb{M}|$.

Formally



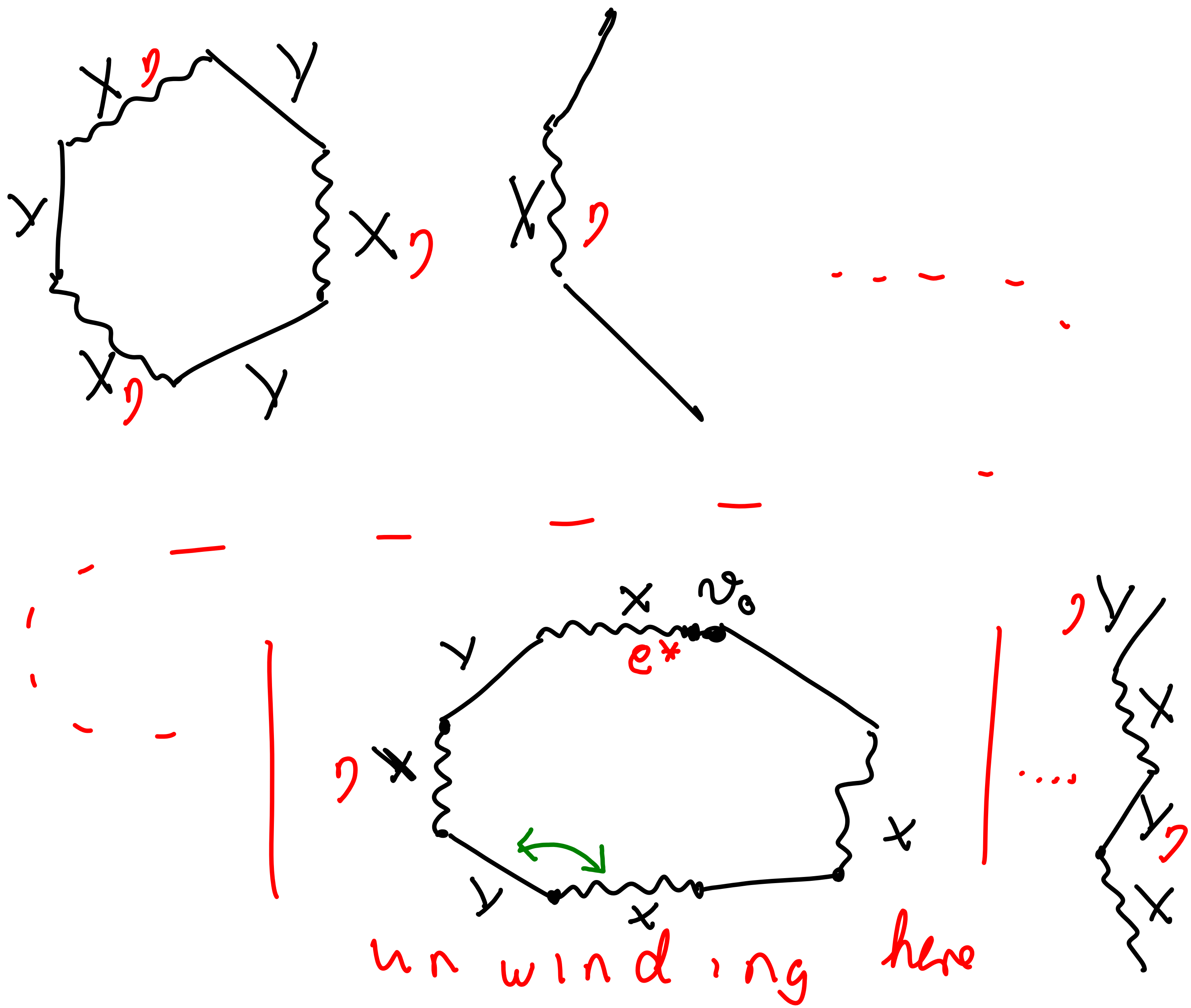
$$\eta_{M,N}(X,Y) = \begin{cases} X \oplus Y \oplus (M \cup N) \\ X \oplus Y \oplus (M \cup N) \end{cases}$$

unwinding a cycle

$(M,N) \in \gamma_{X,Y}$

Claim: $\eta_{M,N}$ is an injection into $\mathcal{M}$.

$$\Rightarrow \quad \theta_{M,N} \leq |\mathcal{M}|$$


X
Y
η
v_0
e*
unwinding here

1) $\eta(X,Y) \in \mathcal{M}$

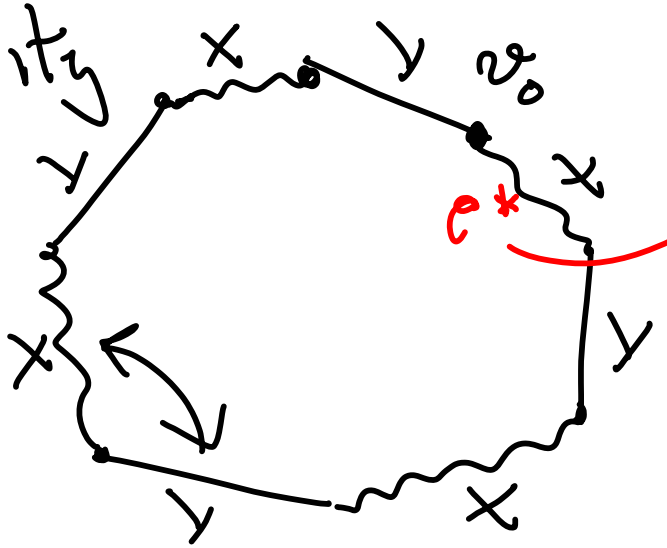
$$A = X \oplus Y \oplus (M \cup N) \subseteq X \cup Y$$

Max degree ≤ 2

Suppose (u, v_1) , (u, v_2) $\in A$

X Y ← Both in $X \oplus Y$
Neither in $M \cup N$

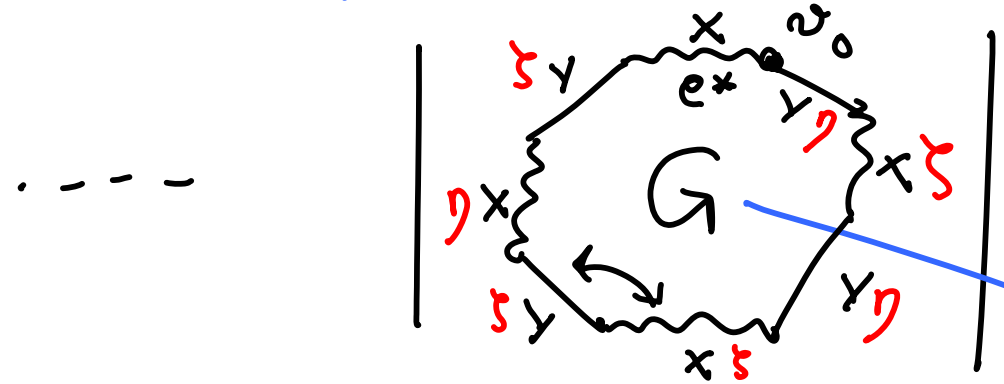
Only possibility



(11) η is injective i.e. we can recover X, Y from η.

$$X \oplus Y = \begin{cases} \eta(X,Y) \oplus (M \cup N) + e^{*} \\ \eta(X,Y) \oplus (M \cup N) \end{cases} \quad \text{or}$$

$$\zeta = \eta(X,Y) \oplus (M \cup N)$$



unwinding in "wrong" direction for path $\Rightarrow$ unwinding cycle.

$$X \cap Y = M - X \oplus Y$$