Department of Mathematical Sciences

21–241 **Matrix Algebra** **Fall 2002**
Professor A. Frieze and Professor S. Ta'asan

Test No. 1

Name:______________________________
Section:____________________________

problem	points	scores
1	25	
2	25	
3	25	
4	25	
total	100	

1. (25 pts) Describe all the solutions to the linear system

$$\begin{array}{rcrcrcr} x_1 & + & 3x_2 & - & 5x_3 & = & 4 \\ x_1 & + & 4x_2 & - & 8x_3 & = & 7 \\ -3x_1 & - & 7x_2 & + & 9x_3 & = & -6 \end{array}$$

$$\begin{bmatrix} 1 & 3 & -5 & 4 \\ 1 & 4 & -8 & 7 \\ -3 & -7 & 9 & -6 \end{bmatrix}$$

4 pts
only 2 pts if RHS left out

$$\begin{bmatrix} 1 & 3 & -5 & 4 \\ 0 & 1 & -3 & 3 \\ 0 & 2 & -6 & 6 \end{bmatrix}$$

4 pts

$$\begin{bmatrix} 1 & 0 & 4 & -5 \\ 0 & 1 & -3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4 pts

Solution

$$\begin{array}{rcrcr} x_1 & = & -5 & - & 4x_3 \\ x_2 & = & 3 & + & 3x_3 \end{array}$$

or

$$\begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

10 pts

3 pt bonus for perfect solution.

2. (25 points) Determine by inspection if the given set is linearly dependent. Give reasons.

a.

$$\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 9 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 8 \end{bmatrix}$$

Linearly dependent. More than 3 vectors in $\mathbb{R}^3$ – 8 pts.

b.

$$\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix},$$

Linearly dependent. The set contains a zero vector – 8 pts

c.

$$\begin{bmatrix} -2 \\ 4 \\ 6 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -9 \\ 15 \end{bmatrix}$$

If dependent, one should be a multiple of the other. Linearly independent – 9 pts.

3. (25 points) Let T be a linear transformation from $\mathbb{R}^n \to \mathbb{R}^m$. Show that if $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$ are linearly dependent then so are $T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_p)$.

There exist $x_1, x_2, \ldots, x_p$ not all zero such that $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_p\mathbf{v}_p = \mathbf{0}$.
5 pts, (3 pts if there is an omission of "not all zero").

Thus $T(x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \cdots + x_p\mathbf{v}_p) = \mathbf{0}$ - 5 pts
Thus by linearity (5pts)
$x_1T(\mathbf{v}_1) + x_2T(\mathbf{v}_2) + \cdots + x_pT(\mathbf{v}_p)) = \mathbf{0}$ - 5 points
and so (5 pts) $T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_p)$ are linearly dependent..

4. (25 points)Mark each statement true or false. No justification is needed.

   2.5 pts per correct answer.

(i) Two matrices are row equivalent if they the same number of rows. **F**

(ii) Two linear systems are equivalent if they have the same solution set. **T**

(iii) Whenever a system has a free variable then the solution set contains infinitely many solutions. **F**

(iv) A linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$ is completely determined by its effect on any n distinct vectors. **F**

(v) A vector $\mathbf{b}$ is a linear combination of the columns of a matric A if and only if the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution. **T**

(vi) If the columns of a p time q matrix A span $\mathbb{R}^p$ then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b}$ n $\mathbb{R}^p$.**T**

(vii) The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution. **T**

(viii) The columns of a matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution. **T**

(ix) Every linear transformation is a matric transformation. **T**

(x) The homogeneous equation is always consistent. **T**