EXAMPLE 4 Given a scalar r, define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = r\mathbf{x}$. T is called a **contraction** when $0 \le r \le 1$ and a **dilation** when $r > 1$. Let $r = 3$ and show that T is a linear transformation.

Solution Let $\mathbf{u}$, $\mathbf{v}$ be in $\mathbb{R}^2$ and let c, d be scalars. Then

$$\begin{aligned} T(c\mathbf{u} + d\mathbf{v}) &= 3(c\mathbf{u} + d\mathbf{v}) && \text{Definition of } T \\ &= 3c\mathbf{u} + 3d\mathbf{v} && \rbrace \text{ Vector arithmetic} \\ &= c(3\mathbf{u}) + d(3\mathbf{v}) && \\ &= cT(\mathbf{u}) + dT(\mathbf{v}) \end{aligned}$$

Thus T is a linear transformation because it satisfies (4). See Fig. 5.

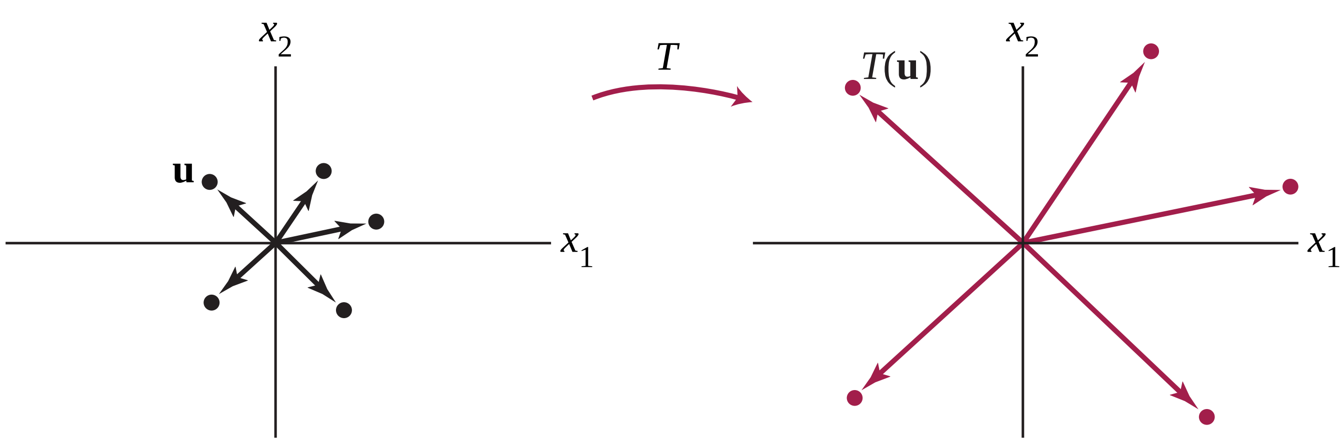

FIGURE 5 A dilation transformation.