EXAMPLE 6′ Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -1 \\ -2 \\ 7 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}$.

(Same $\mathbf{a}_1$, $\mathbf{a}_2$ as in Example 5′.) Determine whether $\mathbf{b}$ is in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$.

Solution Does $x_1\mathbf{a}_1 + x_2\mathbf{x}_2 = \mathbf{b}$ have a solution?
Row reduce $\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{b} \end{bmatrix}$:

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & -2 & 4 \\ -3 & 7 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -3 \\ 0 & -2 & 4 \\ 0 & 4 & -7 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -3 \\ 0 & -2 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

The vector equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 = \mathbf{b}$ has no solution.
So $\mathbf{b}$ is *not* in Span$\{\mathbf{a}_1, \mathbf{a}_2\}$.