Summary:

To study the equation $x_1\mathbf{a}_1 + x_2\mathbf{a}_2 = \mathbf{b}$, consider:

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & -2 & 4 \\ -3 & 7 & 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{b} \end{bmatrix}$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{b} \end{matrix}$$

A vector equation

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n = \mathbf{b}$$

has the same solution set as the linear system whose augmented matrix is

$$\begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n & \mathbf{b} \end{bmatrix} \qquad (*)$$

In particular, **b** *can be generated by a linear combination of* $\mathbf{a}_1, \ldots, \mathbf{a}_n$ *if and only if the linear system corresponding to (*) has a solution.*