

$$\begin{aligned}
 x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 &= \mathbf{b} \\
 x_1 \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ -2 \\ 7 \end{bmatrix} &= \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix} \\
 \begin{bmatrix} x_1 - x_2 \\ 0 - 2x_2 \\ -3x_1 + 7x_2 \end{bmatrix} &= \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}
 \end{aligned}$$

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 x_1 - x_2 &= -3 \\
 0 - 2x_2 &= 4 \\
 -3x_1 + 7x_2 &= 1
 \end{aligned}$$

Solve this system by row reducing the augmented matrix:

$$\begin{bmatrix} 1 & -1 & -3 \\ 0 & -2 & 4 \\ -3 & 7 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -3 \\ 0 & -2 & 4 \\ 0 & 4 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -3 \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

The system is consistent, so \mathbf{b} is a linear combination of \mathbf{a}_1 and \mathbf{a}_2 . To find the weights, complete the row reduction, and obtain $x_1 = -5$, $x_2 = -2$. Thus

$$-5\mathbf{a}_1 - 2\mathbf{a}_2 = \mathbf{b}$$