EXAMPLE 5′ Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$. Determine whether $\mathbf{b}$ is a linear combination of $\mathbf{a}_1$ and $\mathbf{a}_2$. Do weights x_1 and x_2 exist such that

$$x_1\mathbf{a}_1 + x_2\mathbf{a}_2 = \mathbf{b} \tag{1}$$

Solution

$$x_1 \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ -2 \\ 7 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

$\mathbf{a}_1$ $\mathbf{a}_2$ $\mathbf{b}$

Rewrite this vector equation:

$$\begin{bmatrix} x_1 \\ 0 \\ -3x_1 \end{bmatrix} + \begin{bmatrix} -x_2 \\ -2x_2 \\ 7x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$$

and

$$\begin{bmatrix} x_1 - x_2 \\ 0 - 2x_2 \\ -3x_1 + 7x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix} \tag{2}$$