

EXISTENCE AND UNIQUENESS QUESTIONS

EXAMPLE 5

$$\begin{aligned}3x_2 - 6x_3 + 6x_4 + 4x_5 &= -5 \\3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 &= 9 \\3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 &= 15\end{aligned}$$

Solution In Example 3, we obtained the echelon form:

$$\begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix} \quad (x_5 = 4)$$

No equation of the form $0 = 1$, so system **is** consistent.

Free variables: x_3 and x_4 .

Consistent system,
with free variables $\Rightarrow$ infinitely many solutions.

EXAMPLE

$$\begin{aligned}3x_1 + 4x_2 &= -3 \\2x_1 + 5x_2 &= 5 \\-2x_1 - 3x_2 &= 1\end{aligned} \rightarrow \begin{bmatrix} 3 & 4 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{aligned}3x_1 + 4x_2 &= -3 \\x_2 &= 3 \\0 &= 0\end{aligned}$$

Consistent system, no free variables $\Rightarrow$ unique solution.