

■ **EXAMPLE 3'** Is this system consistent?

$$\begin{array}{rcl} 3x_2 - 6x_3 & = & 8 \\ x_1 - 2x_2 + 3x_3 & = & -1 \\ 5x_1 - 7x_2 + 9x_3 & = & 0 \end{array} \quad \begin{bmatrix} 0 & 3 & -6 & 8 \\ 1 & -2 & 3 & -1 \\ 5 & -7 & 9 & 0 \end{bmatrix}$$

Solution Row operations on the augmented matrix:

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & 3 & -6 & 8 \\ 5 & -7 & 9 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & 3 & -6 & 8 \\ 0 & 3 & -6 & 5 \end{bmatrix} \\ \sim \begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & 3 & -6 & 8 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

To interpret this “triangular form” go back to equation notation:

$$\begin{array}{rcl} x_1 - 2x_2 + 3x_3 & = & -1 \\ 3x_2 - 6x_3 & = & 8 \\ 0 & = & -3 \end{array} \quad \leftarrow \text{Never true!}$$