Department of Mathematics
Carnegie Mellon University
21-484 Applied Graph Theory
Test 1

Name:______________________________

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Q1 (25 points)
Find a flow augmenting path relative to the $s-t$ flow in the network below.
Is your new flow maximum? If so, find a minimum capacity $s-t$ cut.

Q2 (25 points)
G is a 2-connected graph i.e. has no cut-points. C_1 and C_2 are both cycles of maximum length in G.
(a) Show that C_1, C_2 have at least one common vertex.
(b)* Show that C_1, C_2 have at least two common vertices.

Q3 (25 points)
A graph $G = (V, E)$ is called *uniquely k-edge colourable* if any two proper k-edge colourings of G induce the same partition of E.
Show that every uniquely 3-edge colourable, 3-regular graph has a Hamilton cycle.
[Hint: take a 3-edge colouring of G and delete all the edges of a single colour. Show that what is left is a Hamilton cycle.]

Q4 (25 points)
Let $n_k = R(3, 3, \ldots, 3)$, [k 3's]. Suppose $n \geq n_k$ and the $2^n - 1$ non-empty subsets of $[n] = \{1, 2, \ldots, n\}$ are each given one of k colours i.e. we are given a mapping $c : \mathcal{P}([n]) \setminus \emptyset \to [k]$. Show that there are $X, Y \subseteq [n]$ such that $X, Y, X \cup Y$ have the same colour.
[Hint: Consider the following colouring of K_n: edge $\{i, j\}, j > i$ is given the colour $c(\{i, i+1, \ldots, j\})$.]