Department of Mathematical Sciences
Carnegie Mellon University

21-484 Graph Theory

Test 2

Name:______________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1 (34 points)
Graph G has a unique odd cycle. Show that $\chi(G) = 3$.

Q2 (33 points)
A plane graph is *outerplanar* if every vertex lies on the outer face. Show that a simple outerplanar graph with $n \geq 2$ vertices has at most $2n - 3$ edges.

Q3 (33 points)
A square 0-1 matrix is *upper-constant* if the elements above the diagonal are all 0 or all 1. Similarly, a square 0-1 matrix is *lower-constant* if the elements below the diagonal are all 0 or all 1. A matrix is *pseudo-constant* if it is both upper and lower constant.
Let M be an $n \times n$ 0-1 matrix. For $S \subseteq \{1, 2, \ldots, n\}$, M_S is the $|S| \times |S|$ matrix where $M_S(i, j) = M(i, j)$ whenever $i, j \in S$.
Show that for any positive integer k, there is an integer n_k, such that if $n \geq n_k$ then for any $n \times n$ 0-1 matrix M there is a set S with $|S| = k$ and M_S pseudo-constant.