Department of Mathematics
Carnegie Mellon University

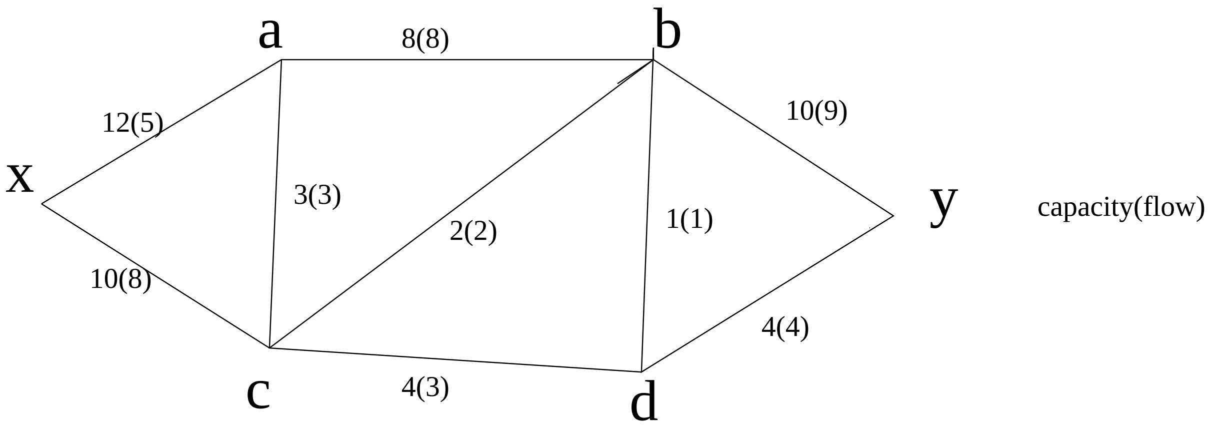
21-484 Applied Graph Theory

Test 3

Name:______________________________

Problem	Points	Score
1	34	
2	33	
3	33	
Total	100	

Q1 (34 points)
Find a flow augmenting path in the network below. Update the flow, prove that your new flow is a maximum x, y-flow and give a minimum cut.



Q2 (33 points)
Let P_n and C_n denote a path and cycle with n edges respectively. Fix a positive integer $k \geq 2$. u_n and v_n are the number of k-colourings of P_n, C_n respectively.
Use the basic recurrence for chromatic polynomials (Theorem 8.6 of Bondy and Murty) to show that

(a) $u_n = (k-1)u_{n-1}$ for $n \geq 2$.

(b) $v_n = u_{n-1} - v_{n-1}$ for $n \geq 3$.

Hence show that (i) $u_n = (k-1)^n k$ and (ii) $v_n = (k-1)^n + (-1)^n(k-1)$.

Q3 (33 points)
A digraph D is *unilateral* if for any two vertices u and v, either v is reachable from u or u is reachable from v. Show that D is unilateral if and only if there is a directed walk that spans all the vertices.
[Hint: consider the tournament T on the same vertex set, where the edge joining u and v is directed according to whether there is a directed path from u to v or v to u.]