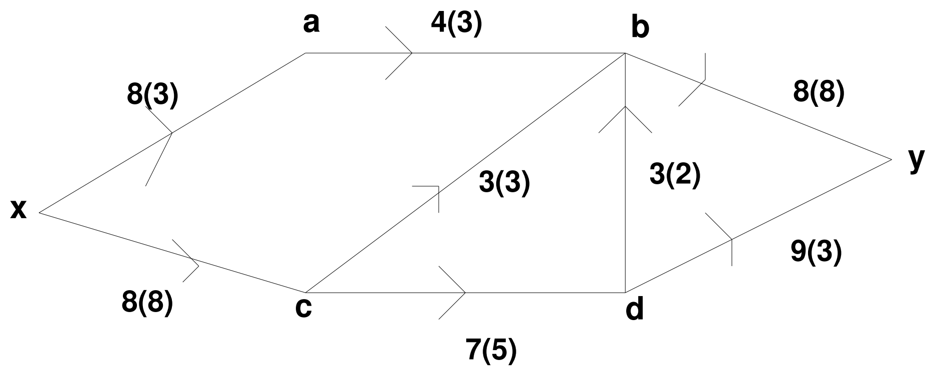
Department of Mathematics
Carnegie Mellon University
21-484 Applied Graph Theory
Test 3

Name:______________________________

Problem	Points	Score
1	34	
2	33	
3	33	
Total	100	

Q1 (34 points)
Starting with the given flow, apply the labelling algorithm to find a maximum value flow and a minimum capacity cut.
Each arc is labelled *capacity (flow)*.



Q2 (33 points)
(a) Show that if G is a planar graph and $\nu \geq 11$ then G^c is non-planar.
(b) Show that if every vertex of a plane connected graph has degree at least 3 then there is a face with 5 or fewer edges.

Q3 (33 points)
A digraph D is *unilateral* if for any two vertices u and v, v is reachable from u or u is reachable from v (or both). Show that D is unilateral if and only if D contains a directed walk which visits each vertex at least once.
[Hint: only if, consider the tournament T with the same vertex set as D and an edge (u, v) if u is reachable from v in D. (In case this would generate both (u, v) and (v, u), choose one arbitrarily.)]