Department of Mathematics
Carnegie Mellon University
21-484 Applied Graph Theory
Test 2

Name:______________________________

Problem	Points	Score
1	34	
2	33	
3	33	
Total	100	

Q1 (34 points)
(a) Let M_1, M_2 be matchings in a simple graph $G = (V, E)$. Suppose that that there is no component of $M_1 \oplus M_2$ which consists of two vertices joined by an edge of M_2. Show that $|M_2| \leq 2|M_1|$.
(b) Let M_1 be a matching of G such the set of vertices *not* covered by M_1 is an independent set. Use the first part to show that if M_2 is *any* other matching of G then $|M_2| \leq 2|M_1|$.

Q2 (33 points)
Let G be obtained from a simple k-regular graph with an odd number of vertices by deleting strictly fewer than $k/2$ edges. Show that $\chi'(G) = k + 1$.

Q3 (33 points)
Show that a graph G is bipartite if and only if $\alpha(H) \geq \nu(H)/2$ for all subgraphs H of G.