Department of Mathematics
Carnegie Mellon University
21-484 Applied Graph Theory
Test 2

Name:____________________________

Problem	Points	Score
1	34	
2	33	
3	33	
Total	100	

Q1 (34 points)
Let A be the adjacency matrix of the graph G. Let $B = A^2$. Suppose now that for all i, j, $B_{i,j} \neq 0$ implies that $A_{i,j} \neq 0$.
Prove that G is a complete graph.
[For a matrix M, $M_{i,j}$ is the entry in row i, Column j.]

Q2 (33 points)
The complement graph $\bar{G}$ of the graph $G = (V, E)$ is the graph with vertex set V and an edge $\{u, v\}$ iff $\{u, v\} \notin E$ e.g. if G is a complete graph then $\bar{G}$ has no edges and conversely.
Prove that

$$\chi(G)\chi(\bar{G}) \geq |V|,$$

where as usual, χ denotes chromatic number.

Q3 (33 points)
$G = (V, E)$ is a connected graph. G does not have a perfect matching, but $G \setminus v$ has a perfect matching for all $v \in V$.
Prove that G has no bridges.