Department of Mathematical Sciences
Carnegie Mellon University

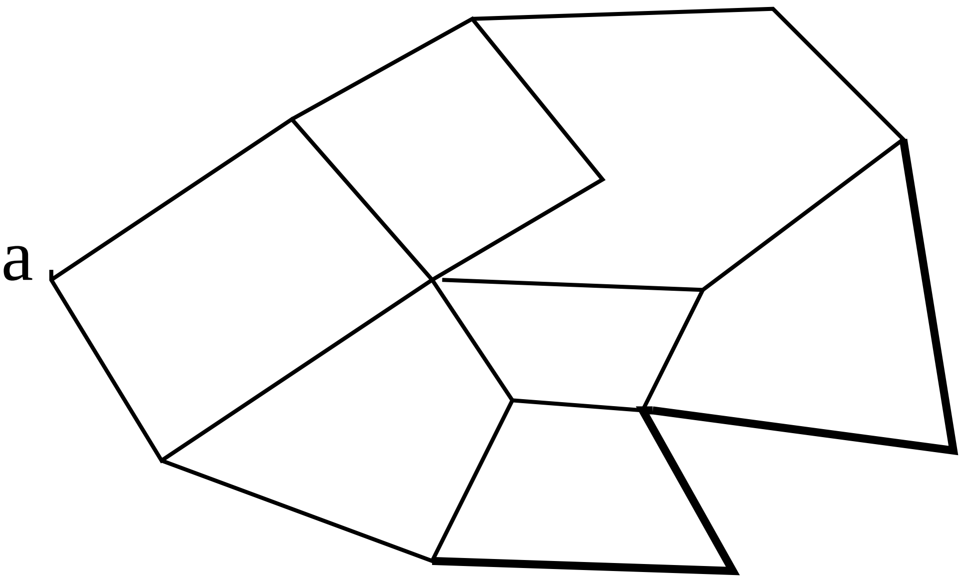
21-484 Graph Theory

Test 1

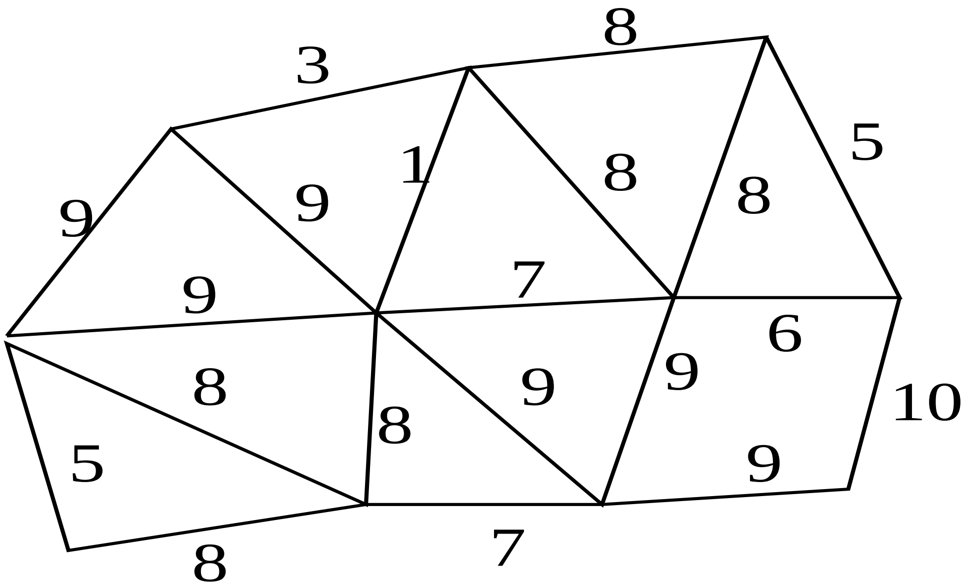
Name:____________________________

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

Q1 (25 points) **(a)**
Find a proper 2 colouring of the vertices of the following graph by using Breadth-First-Search from vertex a:



(b) Find a maximum weight spanning tree of the following graph:



Q2 (25 points) Let P be a path of length $k \leq \nu - 2$ in a connected graph G. Suppose that P has endpoints u, v and $e = uv$ is an edge of G. Show that G has a path of length $k + 1$.
(Hint: there is at least one edge joining the cycle $P + e$ to the rest of the graph.)

Q3 (25 points) Let P_1, P_2 be paths of maximum length in a connected graph G. Show that they share a common vertex.
(Hint: Proof by contradiction: Assume that P_1, P_2 are vertex disjoint. Show that G must contain a path Q from P_1 to P_2 whose non-endpoint vertices are disjoint from P_1, P_2. Use Q to get a longer path than P_1, P_2.)

Q4 (25 points) A tree has n vertices all of which have degree 1 or d for some integer $d \geq 2$. Suppose n_1 are of degree 1 and n_d are of degree d. Show that

$$n_d = \frac{n-2}{d-1}.$$

(Hint: write down 2 linear equations involving n_1 and n_d or use induction.)