Department of Mathematics
Carnegie Mellon University

21-484 Applied Graph Theory

Test 1

Name:____________________________

Problem	Points	Score
1	34	
2	33	
3	33	
Total	100	

Q1 (34 points)
The simple graph G has maximum degree $\nu - 2$ and diameter 2. Prove that it has at least $2\nu - 4$ edges. [Diameter 2 means that there is a path of length 2 joining any 2 non-adjacent vertices.]
[Hint: Take a vertex x of degree $\nu - 2$ and let y be the unique non-neighbour of x. Let N be the set of neighbours of x and M be the set of neighbours of y. Show that each $v \in N \setminus M$ has a neighbour in M. Now count the number of edges you know exist.]

Q2 (33 points)
The graph F_n consists of n 4-cycles sharing a common vertex. It has $3n + 1$ vertices and $4n$ edges and F_4 is drawn below. Use the matrix tree theorem to show that F_n has 4^n spanning trees.

Q3 (33 points)
The n-cube C^n has vertex set $\{0,1\}^n$ and an edge joining a pair of vertices $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots y_n$ iff there is some t such that $x_i = y_i$ if $i \neq t$ and $x_t = 1 - y_t$. The 3-cube C^3 is drawn below.
Show that C^n is Hamiltonian for all $n \geq 2$.
[Hint: observe that C^n can be viewed as the union of 2 copies of C^{n-1}, by considering the subgraphs induced by the vertices with $x_n = i, i = 0, 1$. Prove the result inductively.]