Department of Mathematics
Carnegie Mellon University

21-484 Applied Graph Theory

Test 1

Name:______________________________

Problem	Points	Score
1	34	
2	33	
3	33	
Total	100	

Q1 (34 points)
A connected graph G has minimum degree $\delta \geq 2$ and has no cycles of length less than g. Prove that it contains a path of length

$$\min\{g + (\delta - 2)(g - 2) - 1, \nu - 1\}.$$

[Hint: take a path $v_1, v_2, \ldots, v_k$ of maximum length and consider which vertices can be neighbours of v_1.]

Q2 (33 points)
The graph F_n consists of n triangles sharing a common vertex. It has $2n+1$ vertices and $3n$ edges and F_6 is drawn below. Use the matrix tree theorem to show that F_n has 3^n spanning trees.

Q3 (33 points)
The n-cube C^n has vertex set $\{0,1\}^n$ and an edge joining a pair of vertices $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots y_n$ iff there is some t such that $x_i = y_i$ if $i \neq t$ and $x_t = 1 - y_t$. The 3-cube C^3 is drawn below.
Show that C^n is Hamiltonian for all $n \geq 2$.
[Hint: observe that C^n can be viewed as the union of 2 copies of C^{n-1}, by considering the subgraphs induced by the vertices with $x_n = i, i = 0, 1$. Prove the result indictively.]