Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics, Spring 2002

Optional question on the pigeonhole principle

Let $m = n^4 + 1$. Given two sequences $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_m$ of real numbers, show that there exist $i_1, i_2, \ldots, i_{n+1}$ such that *both* subsequences $a_{i_1}, a_{i_2}, \ldots, a_{i_{n+1}}$ and $b_{i_1}, b_{i_2}, \ldots, b_{i_{n+1}}$ are monotone.
Here we do not insist that both are monotone increasing or both are monotone decreasing, so one can be increasing and the other decreasing.
(Hint: There is a long monotone subsequence of the a-sequence. Consider the subsequence of the b-sequence with only the corresponding indices.)