Department of Mathematics
CARNEGIE MELLON UNIVERSITY

Discrete Mathematics 21-228

Answers to homework 8.

Q1: Suppose G is disconnected. Then the vertex set can be partitioned into 2 non-empty sets A and B such that there are no edges between A an B. (A, B will be the union of components). Let $|A| = m$. Then G has at most

$$\binom{m}{2} + \binom{n-m}{2} \leq \frac{n^2 - 3n + 2}{2} \tag{1}$$

edges. The LHS of (1) is maximised when $m = 1$ or $n - 1$.

Q2: If G is bipartite then every subgraph H of G is bipartite. Let S, T be the bipartition of H with $|S| \geq |T|$. Then S contains no edges of H.
If G is not bipartite then G contains an odd cycle C, say. If C has $2k + 1$ vertices then every $k + 1$ vertices of C contain at least one edge.

Q3: If G s disconnected. Then the vertex set can be partitioned into 2 non-empty sets A and B such that there are no edges between A an B. Thus G^c contains the complete bipartite graph with A and B as vertex sets. This bipartite graph is connected – (i) each $a \in A$ is connected to each $b \in B$ by an edge, (ii) if $a_1, a_2 \in A$ then they are connected by a path (a_1, b, a_2) where b is any vertex of B.