Discrete Mathematics 21-228: Spring 2002
Homework.
HW5 due Wednesday 2/27/02

Q1: A clown stands at the edge of a swimming pool, holding a bag containing n red and n blue balls. He draws a ball out one at a time and discards it. If he draws a blue ball he takes one step back; if a red ball, one step forward. (All steps have the same size). What is the expected number of steps until either the clown runs out of balls or falls into the water?

Q2: Solve the recurrence

$$a_n - 7a_{n-1} + 12a_{n-2} = 0 \qquad n \geq 2.$$

$a_0 = a_1 = 1$.

Q3: Show that the number of binary sequences which do not contain any subsequence of the form 101 satisfies the recurrence $c_0 = 1$, $c_1 = 2$, $c_2 = 4$ and $c_n = c_{n-1} + c_{n-3} + c_{n-4} + \cdots + c_0 + 2$ for $n \geq 3$.
[Hint: consider the 1's at the start of the sequence.]