Department of Mathematics
CARNEGIE MELLON UNIVERSITY

Discrete Mathematics 21-228

Answers to homework 1.

Q1 Think of each die as a ball and each outcome as a colour. Thus we want the number of ways of colouring m balls with 12 colors i.e. $\binom{m+11}{11}$. Equivalently, the number of unordered collections of m objects from a set of size 12.

Q2 Each term is of the form $x_1^{t_1} x_2^{t_2} \cdots x_n^{t_n}$ where $t_1 + t_2 + \cdots + t_n = m$. The number of terms equals the number of ways of choosing the t_i's which is $\binom{n+m-1}{n-1}$.

Q3 Add the 0's at each end. Then we have x_1 0's followed by a 1, followed by x_2 0's followed by a 1, ... where $x_1 + \cdots x_{q+1} = q + 2$ and $x_j \geq 1$ for $j = 1, 2, \ldots, p + 1$. Thus the number of strings is $\binom{q+1}{p}$.

Alternatively, lay down the q zeros in a line and choose p positions from the $q - 1$ gaps between them and the 2 positions before and after them.

Alternatively (D.K.) add a 0 to the right hand end of the sequence. Now we have a sequence of 10's and 0's arranged in any order. There are p 10's and $q + 1 - p$ 0's, so $\binom{p+q+1-p}{p}$ possible choices.