Department of Mathematics
CARNEGIE MELLON UNIVERSITY

Discrete Mathematics 21-228

Answers to homework 11.

Q1: Suppose G has a perfect matching M. B's strategy is to choose v_{2i} to be the vertex which is matched to v_{2i-1} in M. Thus if A has a move, so has B.

If G has no perfect matching, let M be a maximum matching and let v_1 be any M-unsaturated vertex. Assume inductively that A has forced the sequence $v_1, v_2, \ldots, v_{2i-1}$ to be M-alternating. This is true for $i = 1$. If $i > 1$ then the edge (v_{2i-2}, v_{2i-1}) is in M. So unless B has lost, B must choose an edge (v_{2i-1}, v_{2i}) which is not in M. v_{2i} is not M-unsaturated, else the current path is augmenting. So A will be able to choose an M-edge (v_{2i}, v_{2i+1}) completing the induction. Note that we have shown that A has a move.

Q2: If a tree T has two distinct matchings M and M' then $M \Delta M'$ consists of a non-empty collection of vertex disjoint cycles i.e. T contains a cycle – contradiction.

Q3: We prove this by induction on k. It is trivially true for $k = 1$ since the graph G is simply a matching. If $k > 1$ then G contains a perfect matching M and $G' = G - M$ is $k-1$-regular. The inductive hypothesis implies that G' is the union of $k - 1$ disjoint perfect matchings $M_1, M_2, \ldots, M_{k-1}$. Thus G is the union of disjoint matchings $M_1, M_2, \ldots, M_{k-1}$ and M.