Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics: Section 1
Test 4

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
Let G be a bipartite graph with vertex partition A, B where $|A| = |B| = n$. Assume that every vertex of G has degree at least $n/2$. Let M be a matching of G which is not perfect. Show that there is an M-augmenting path of length 1 or 3.

Q2: (33pts)
Show that if the edges of K_6 are coloured Red and Blue, then there are at least **two** triangles all of whose edges are of the same colour.

Q3: (34pts)
Show that a digraph $D = (V, A)$ is strongly connected iff for every $S \subseteq V$, $S \neq \emptyset, V$, there is at least one arc (x, y) where $x \in S$ and $y \notin S$.
[Hint: consider what is reachable from a fixed vertex.]