Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics
Test 4

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
A bipartite graph G has bipartition A, B. Let X be the set of vertices of maximum degree in G. Show that G has a matching which saturates $X \cap A$.

Q2: (33pts)
Let T be a tournament. Let x, y be vertices of T such that there is no **directed** path from x to y which uses 1 or 2 arcs. Show that $d^+(y) > d^+(x)$.

Q3: (34pts)
Let G be a connected graph with n vertices. Show that it has a unique cycle if and only if it has n edges.