Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics, Fall 1999: Test 3

Name:______________________________________

Please circle your TA's name: Ojas Parekh   Goran Konjevod

Problem	Points	Score
1	34	
2	33	
3	33	
Total	100	

No notes, tables, books, calculators can be used during the test!

Q1: (34pts)

(a) A connected graph G has two paths P_1, P_2 of lengths k_1, k_2 respectively. Show that if P_1 and P_2 are vertex disjoint then G contains a path of length at least $\lceil k_1/2 \rceil + \lceil k_2/2 \rceil + 1$.

(b) Using (a), show that if G is a connected graph and P_1, P_2 are paths of *maximum* length then they share at least one common vertex.

Q2: (33pts)
Show that a connected graph G satisfying any one of the following conditions is necessarily a tree:

1. Every edge of G is a cut-edge.
2. For every pair of vertices u, v there is exactly one path joining them.

Q3: (33pts)
A digraph D is *unilateral* if for every pair of vertices u, v there is a directed path from u to v or a directed path from v to u or both. Show that D is unilateral if and only if it has a directed walk which goes through every vertex.
(Hint: D is not necessarily a tournament, but you can use the fact that every tournament contains a directed hamilton path.)