Department of Mathematics
Carnegie Mellon University

21-228 Discrete Mathematics: Section 1
Test 3

Name:____________________________________

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
$G = (V, E)$ is a connected graph. G does not have a perfect matching, but $G \setminus v$ has a perfect matching for all $v \in V$.
Prove that G has no cut edges.

{Hint: there is a simple answer based on parity.}

Q2: (33pts)
Let q be a positive integer and $Q = \binom{2q}{q}$. Assume that each edge of K_Q has been coloured Red or Blue so that there is no monochromatic K_{q+2}. Show that there is a K_{q+1} of one colour and a K_q of the other colour.

{Hint: $Q = \binom{(q+1)+(q+1)-2}{(q+1)-1}$ and $Q > \binom{(q+2)+q-2}{(q+2)-1}$. Note the careful way of writing these binomial coefficients.}

Q3: (34pts)
A r-regular graph $G = (V, E)$ is called *M-partitionable* if its edges E can be partitioned into r perfect matchings. We say that G is *uniquely M-partitionable* if E has a *unique* partition into r perfect matchings.

(a) Show that if $r = 2$ and G is uniquely M-partitionable then G is a cycle.

(b) Show by induction on r that if $r \geq 2$ and G is uniquely M-partitionable then G is Hamiltonian.